COVARIANCE REALIZATION PROBLEM FOR SPIN SYSTEMS

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Abstract. We consider a covariance realization problem for spin systems. Let $\Omega_n = \{-1, 1\}^n$ be the space of n-length sequences which are denoted by $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$, where each $\sigma_i \in \{-1, 1\}$ is called a spin and we refer to this space as a spin system. Given a probability on Ω_n and the spin-spin correlations, $c_{ij} := E(\sigma_i \sigma_j)$ (we assume $E(\sigma_i) \equiv 0$ for all $1 \leq i \leq n$), under what conditions does a distribution with those correlations exist, and how does one determine it? Note that, we are not assuming that all the $c_{ij}'s$ are given.

The necessary and sufficient conditions for a covariance matrix of order $n \leq 4$ to be a spin covariance matrix are already known. We give a fairly general and large set of inequalities, parametrized by $\lambda \in M_n(\mathbb{R})$, that are necessary and sufficient for any n. We also give a minimal set of necessary and sufficient conditions for $n = 5, 6$. Finally, we discuss methods to explicitly find the measure that realizes the given spin correlations (if they are feasible). We give a deterministic algorithm as well as a stochastic version of the same algorithm to find the measure explicitly.

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