Interdisciplinary programme in

**INDUSTRIAL ENGINEERING & OPERATIONS RESEARCH**
**INDIAN INSTITUTE OF TECHNOLOGY BOMBAY**

Sample Questions for M.Tech. Admissions Entrance Test
(some of which appeared in 2006 paper)

**Instructions**: *No clarifications on the questions should be sought during the examination. Calculator not required*

1. The Normal probability distribution is also called ________________ (in honor of the person who proposed it as a model for statistical measurement errors)

(P) Gaussian Distribution (Q) Student's t Distribution
(R) Bernoulli Distribution (S) Poisson Distribution

2. If the probability of head appearing in a single toss of a coin is $p$, then the probability that head appears for the first time in the $10^{th}$ toss is:

(P) $p(1-p)$^9 (Q) $p$^10
(R) $p(1-p)$ (S) $(1-p)$ $p$^9

3. A and B working together can finish a job in T days. If A works alone and completes the job, he will take T + 5 days. If B works alone and completes the same job, he will take T + 45 days. What is T?

(P) 25 (Q) 60 (R) 15 (S) None of these

4. Let random variable X and Y have probability mass as shown in the table on the right side. For example, P(X=3, Y=0) = 0.2.

| Y \ X | 1 | 2 | 3 |
|---|---|---|---|
| **0** | 0.1 | 0.2 | 0.2 |
| **2** | 0.3 | 0.2 | 0 |

(i) P(X = 1) = ____________________
(ii) $P(X \leq 2 \mid Y = 0)$ = ____________________
(iii) E[X | Y = 0] = ____________________
(iv) Let Z = *min*(X, Y). Now, E[Z] = ____________________

5. Pick the random variable with least variance:

(P) X is Bernoulli with parameter 0.5 (Q) X is uniform over interval [0, 1]
(Q) X is exponential with rate 1 (S) X = 100

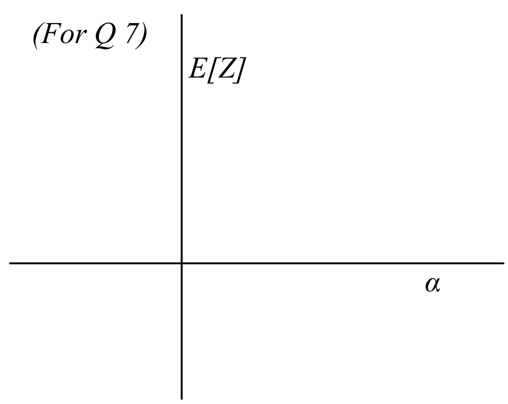
6. The maximum value of solution of ordinary differential equation $\frac{d^2 f(x)}{dx^2} = f(x)$

with $f(0) = 1$ is ________________.

7. Let $X$ and $Y$ be two random variables with E[X] = 2 and E[Y] = 3. In the space given on the right side, **plot E[Z]** as a function of $\alpha$ where,

$Z = \alpha X + \beta Y$ for reals $\alpha$ and $\beta$ such that $\alpha + \beta = 1$.

*(For Q 7)*




8. Let $A$ be a $N$ x $N$ matrix with each element $\frac{1}{N}$. Now, it is true that:

(P) Zero is an eigenvalue of $A$
(Q) Determinant of $A$ is non-zero
(R) Determinant of $A$ is zero
(S) Determinant depends on value of $N$

9. For any random variable X, it is true that:

(P) $E[X^2] \geq (E[X])^2$ (Q) $E[X^2] < (E[X])^2$
(R) $E[X^2] < 0$ and $(E[X])^2 \geq 0$ (S) Such relations depend on the random variable X

10. Find the maxima and/or the minima of the function $f(x) = x^3 + 3x^2 - 24x + 3$.

11. For the following pseudo-code, write the entire output when $n = 10$: (Note: *the write() function prints the value of its parameters on the screen*)

```
x = 0;  y = 1;
write(x, y);
while (n != 0)
{
        f = x + y;
        write(f);
        n--;
         x = y;
        y = f;
}
```

12. Consider the Linear Programming model on the right side:

(i) Identify the solution space using a graph that defines all the feasible solutions of the model.
(ii) For the given objective, identify the corner point(s) that define the optimum solution.

$$\begin{aligned} &Max \quad z = 5x + 4y \\ &subject\ to \\ &6x + 4y \le 24 \\ &x + 2y \le 6 \\ &-x + y \le 1 \\ &y \le 2 \\ &x, y \ge 0 \end{aligned}$$

13. For the system of linear equations Ax = b for a square, non-singular matrix A (of dimension n), which of the following are true?

(P) There is a unique solution of this system for any vector b
(Q) There is a non zero solution for any non zero vector b
(R) There is no solution for the case b = 0
(S) There are infinitely many solutions for any vector b

14. The function of two variables $f(x,y) = x^2 - y^2$ over $\mathbb{R}^2$ has

(P) A local minimum and a local maximum, but no global minima or maxima
(Q) No local minimum or local maximum
(R) No stationary point (where the gradient vector is zero)
(S) One global minima and one local maxima

15. The transportation problem in linear programming is of the form

$$\begin{aligned} &\text{Min } \Sigma_i \Sigma_j c_{ij} x_{ij} \\ &\text{s.t.} \quad \Sigma_j x_{ij} = a_i \text{ for i from } 1, \ldots, m \\ &\qquad \Sigma_i x_{ij} = b_j \text{ for j from j} = 1, \ldots, n, \\ &\qquad \text{all } x_{ij} >= 0. \end{aligned}$$

If the transportation problem has an optimal solution, then

(P) The maximum number of non zero $x_{ij}$ values is m
(Q) The maximum number of non zero $x_{ij}$ values is n
(R) The maximum number of non zero $x_{ij}$ values is m+n
(S) The maximum number of non zero $x_{ij}$ values is m+n-1

16. With respect to the assignment problem in linear programming, which of the following is true?

(P) The assignment problem is a special case of the transportation problem
(Q) The transportation problem is a special case of the assignment problem
(R) Neither the transportation nor the assignment problems are special cases of the other
(S) The assignment problem will result in a degenerate solution for the relevant LP

17. The problem: Max $xyz$ s.t. $x + y + z = 10$, $x, y, z \ge 0$ has

(P) No feasible solution
(Q) A unique solution
(R) Multiple optimal solutions
(S) Unbounded solution (i.e. no optimal solution)

18. Let $A$ be the optimal objective function value for the problem $\min f(x)$ s.t. $g_1(x) \le 0$ and
$B$ be the optimal objective function value for the problem $\min f(x)$ s.t. $g_1(x) \le 0$, $g_2(x) \le 0$,
for some real-valued functions $f$, $g_1$ and $g_2$ of $\mathbb{R}^n$. Assume that there is some $x$ that satisfies $g_1(x) \le 0$ and $g_2(x) \le 0$. Then

(P) A = B (Q) $A \le B$
(R) $A \ge B$ (S) Not possible to conclude any of the above.

19. Suppose the stock price that is $S$ now becomes $S*u$ with probability 0.6, or $S*d$ with probability 0.4 after one week for given reals $u$ and $d$. Price fluctuations from first week to second week have same probability distribution and independent of those in first week. Take $S = 100$, $u = 1/d = 1.1$.

(i) What is the mean of stock price after second week?

(ii) The variance of stock price after second week = ____________.

(iii) Suppose you bought 1 unit of stock at the beginning of first week and you <u>don't</u> want to sell the stock after second week if the prevailing price is less than Rs.100. The return is $max\{S_2 - 100, 0\}$ where $S_2$ is the price after second week. What is the mean return?