IE 601, Optimization Techniques

Assignment 01, August 15, 2019

Due Wednesday, August 21, 2019 in class

Note: There are 5 questions on 1 page(s). Submit a report written in your own words. Write your name and roll number clearly on the report.

1. Convert the following linear program into standard form:

$$\begin{aligned}
\min_x \; & 4x_1 + x_2 - 3x_3 + x_4 + x_5 \\
\text{s.t. } & 3x_1 + x_2 \leq 4, \\
& x_1 + x_2 + x_3 \geq -18, \\
& x_4 + 100x_5 = 67, \\
2 \leq {} & 5x_2 + x_3 - 4x_4 \leq 10, \\
-20 \leq {} & x_1 \leq 50, \\
& x_2 \leq 0, \\
2 \leq {} & x_3 \leq 20, \\
0 \leq {} & x_4 \leq 1.
\end{aligned}$$

2. For each of the following sets, explain whether it is convex or not.
   (a) $\{(x, y) \in \mathbb{R}^n \times \mathbb{R} : f(x) \leq y\}$, where f is a given convex function
   (b) $\left\{x \in \mathbb{R}^4 : \sqrt{x_1^2 + x_2^2 + x_3^2} \leq x_4, x \geq 0\right\}$
   (c) $\left\{x \in \mathbb{R}^4 : x_1 x_2 + x_3 x_4 \geq 1, x \geq 0\right\}$
3. Exercise 2.39 [BJS, 2nd Ed.]
4. Find all extreme points and extreme rays of the set $\left\{x \in \mathbb{R}^4 : 3x_1 + 2x_2 + x_3 + 4x_4 \geq 12, x_1 - x_2 \geq 1, x \geq 0\right\}$
5. Consider a polyhedral region, P in $\mathbb{E}^n$ defined by the following constraints

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, \ldots, n, \tag{1}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, \ldots, n, \tag{2}$$

$$0 \leq x_{ij} \leq 1, \quad i = 1, \ldots, n, j = 1, \ldots, n. \tag{3}$$

   Show that each constraint defined by (1) and (2) is redundant individually. How many of them can be removed without affecting the polyhedral region P?