IE 601, Optimization Techniques

Assignment 06, October 28, 2019

Due Monday, November 04

Note: There are 6 questions on 1 page(s). Submit a report written in your own words. Write your name and roll number clearly on the report.

1. Show that the matrix $uu^\intercal$, where u is a vector in $\mathbb{R}^n$ has rank 1.

2. Consider the expressions for updating B^{k+1} and H^{k+1} in a BFGS iteration. Assuming B is the inverse of H, check whether B^{k+1} is an inverse of H^{k+1}. Also check whether BFGS update in H^{k+1} is a rank-2 update. Check also whether the update in B^{k+1} is of rank-2.

3. Draw the contours of the function $5x_1^2 - 2x_1 + 4x_1x_2 + x_2^2$. Find a non-optimal point where the steepest descent and Newton directions are the same. Find another point where they are different.

4. Starting at the point $[0 \;\; 0]^\intercal$, solve the above problem using BFGS. You may use exact line search to find the step length.

5. Transform the above problem into a sum of squares of the two affine functions f_1 and f_2 of x. Substitute $y_1 = f_1$ and $y_2 = f_2$. Note that we are making an affine transformation $y = Ax + b$. If we minimize $y_1^2 + y_2^2$, is it 'equivalent' to minimizing f? Compare the directions of steepest descent and Newton's method (only first iteration) for this problem to those of the above problem. Use the starting point $x = [0 \;\; 0]^\intercal$.

6. Exercise 3.11 [Fletcher]