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https://doi.org/10.1007/s12597-021-00567-9

APPLICATION ARTICLE

# Multi-modal supply chain distribution problem

**Hussain A. Kharodawala[1] · Ashutosh Mahajan[1] · Jayan Moorkanat[2]**

Accepted: 8 September 2021



## Abstract

Supply chain networks are representation of interaction among different entities. Usually these entities are facilities which can be represented as nodes in a network and the flow of material between them can be represented as flow on arcs (paths) connecting them. These flows can be facilitated via multiple modes available to transport material from one facility to another. We discuss a multi-modal supply chain distribution problem where the aim is to minimize sum of transportation cost on various modes between facilities, inventory, backlog and lost sales costs over a time-horizon. The problem can be represented as a time-space network of nodes and arcs. Each node defines the state of a facility at a given time-period and the arcs between these nodes are either transportation, inventory or backlog carrying arcs. The time-horizon consists of discrete time-periods and the flows on transportation arcs are required to be an integer multiple of predefined lot sizes as in vehicle capacities, batch sizes, etc. Apart from this, there are certain business rules which are posed on transportation modes incoming to a facility or posed on the suppliers of a facility are to be followed. The problem stated above is first modeled as a Mixed Integer Linear Program (MILP) and solved using a MILP solver. We propose integer rounding heuristics to get a feasible solution to the problem. We report in our results that these heuristics can be used to generate an integer feasible solution quickly. Using this feasible solution as an MIP start in solver helps us in reaching optimal solution in lesser time.


**Keywords** Supply chain · Heuristics · Modeling

✉ Hussain A. Kharodawala
hussain.abbas52@gmail.com

Ashutosh Mahajan
amahajan@iitb.ac.in

Jayan Moorkanat
jayan@spashtatech.com

1 IEOR, IIT Bombay, Powai, Mumbai, India

2 Spashta Technologies, Bengaluru, India

Published online: 11 October 2021

## 1 Introduction

A supply chain [2] consists of a set of entities involved in fulfilling customer demand for products and/or services. These entities could be manufacturers, warehouses, retailers, transporters, customers, etc. They are connected with each other for exchange of material and information. The objective of supply chain as a function is to fulfil the end customer demand at the right time and at an affordable cost. Various costs such as manufacturing, procurement, transportation, inventory, etc. are incurred to achieve this objective. An efficient supply chain attempts to operate at optimal level of these costs so as to gain maximum benefit possible for the entities involved.

Logistics (also referred as transportation) [15], an important function of a supply-chain involves the planning of movements of materials/goods from source (suppliers, factories, warehouses, etc.) to point of consumption either directly or via intermediate stops facilitated using different modes of transport. This is required to ensure that the materials are delivered timely in required quantities so that the resources using them are well utilized and the consumers demanding them are satisfied. An efficient transportation plan impacts not only the transportation costs but also inventory costs, costs due to late deliveries and cost of the material being transported. To face uncertainties and shortages, organizations introduce multiple echelons in their supply chains such as warehouses, distributions centers which hold inventory and act as a buffer against the shock of uncertainties and shortages. Also, supply chain may be multi-echelon inherently because there might not be a direct path for a source and destination pair.

Logistics planning can be manual or be done by tools that rely on either some heuristics, mathematical models, or exact algorithms. Manual planning often relies on the business acumen of the planner and is convenient if the data required in planning is humanly manageable. It is not robust to frequent changes in data and re-planning or doing what-if scenarios could be tedious. Heuristics is an approach where first the objective and requirements to be fulfilled by the plan are well identified and the plan is generated to satisfy these requirements by defining a process or series of steps to make a plan. The heuristics used for planning can be ad hoc to the problem or based on popular approaches like Genetic Algorithm [8], Tabu Search [7], Simulated Annealing [17] among others. Heuristics can be quite effective in generating good plans within less computational time, but face difficulty of ensuring that the plan is optimal.

The planning can also be done by abstracting the problem as a mathematical model that that fits some general mathematical paradigm that can be solved by available solvers for them or by developing an exact algorithm. Using such an approach can guarantee an optimal or near optimal plan with an estimate of how far the plan is from optimality. Some examples of mathematical model categories are Linear Programs(LP), Nonlinear Programs(NLP), Mixed Integer Programs(MILP), Mixed Integer Nonlinear Programs(MINLP). If plan is modelled as a Linear Program (LP), then the planning can be done in polynomial time. It seldom happens that the model is LP and generally, the planning involves some discrete choices of decision variables to be made which makes the planning problem fall under the category of Mixed Integer Linear Program (MILP) or Mixed Integer Non Linear Programs (MINLP) which are NP-hard.

In this article, we discuss a multi-modal supply chain distribution problem in multi-echelon network where the aim is to minimize sum of transportation cost on various modes between facilities, inventory, backlog and lost sales costs over a time-horizon. The problem can be represented as a time-space network of nodes and arcs. Each node defines the state of a facility at a given time-period and the arcs between these nodes are either transportation, inventory or backlog carrying arcs. The time-horizon consists of discrete time-periods and the flows on transportation arcs are required to be an integer multiple of predefined lot sizes as in vehicle capacities, batch sizes, etc. Apart from this, there are certain business rules which are posed on transportation modes incoming to a facility or posed on the suppliers of a facility.

First, we model the problem as a MILP [3] and solve it using a MILP solver [4]. We propose integer rounding heuristics to get a feasible solution to the problem. While MILP solvers have several heuristics developed for general purpose problems [1], they sometimes can not gain insights from the special structure of the model like we do here. We report in our results that these heuristics can be used to generate a feasible solution quickly. Using this feasible solution as an MIP start (initial solution) in solver helps us in reaching optimal solution in lesser time. When the solution given by integer rounding heuristics is used as a starting point for the solver, it helps the solver to reach optimal solution or solution with reduced gap from the optimal solution in lesser time.

Section 2 provides the literature survey of multi-modal supply chain planning in various domains and methods used for planning. The multi-modal supply chain distribution problem under consideration and MILP model is explained in sect. 3. Section 4 shows how the model can be visualised as a network. Section 5 discusses the proposed rounding heuristics to generate a feasible solution for the problem. Section 6 discusses the results of computational experiments conducted and conclusions drawn from them.

## 2 Literature review

Multi-modal supply chain problems are defined as problems where there are multiple modes connecting two facilities such as rail, road, ocean, etc. or the route from one facility to another is reached after a sequence of modes of transport and we select a mode to transport commodities subjected to constraints such as arc capacities, cost of transportation, availability, etc. Udomwannakhet et al. [16] provides a review on multi-modal transportation models.

Haghani and Oh [10] presents a MILP model to address multi-commodity, multi-modal logistics with time horizon in disaster management and proposes two heuristics algorithms. A common characteristic of their model and ours is the availability of multiple modes, linear cost structure, deterministic supply/demands. We differ on the fronts of number of commodities (one in our case), consideration of transfer times of modes at facilities (not considered here) and additional business rules.

In terms of objective, they have considered vehicular and commodity transportation costs separately in addition to inventory/backlog costs and mode transfer costs. The flows in the network under consideration are supply/demand carryover, transportation via a mode from one facility to another and transfer of material from one mode to another at a facility. The two algorithms discussed by them, one is based on decomposition of the problem and Lagrangian relaxation and other is a fix-and-run heuristic. The complicating constraints are the constraints binding vehicular and commodity flows due to multi-commodity aspect of the problem. The relaxation of these constraints reveals a block structure and the problem is decomposed into two sub-problems. In the fix-and-run heuristic, they iterate over time-periods in the time-horizon and fix values of variable to rounded integer pertaining to a time and solve LP relaxation. After solving a series of LPs, they are able to get an integer feasible solution for the MILP model. The heuristics we propose take inspiration from the fix-and-run heuristic.

Crainic and Rousseau [5] addresses multi-modal, multi-commodity problem in freight transportation for service network design, traffic routing and determination of terminal policies presenting a general modeling framework using a network optimization model to reduce delays and operating costs. They propose a column generation algorithm for their model.

Crainic et al. [6] developed an optimization model for multi-mode, multi-product network which has the network assignment method implemented in an interactive-graphic system for the strategic analysis and planning of freight transportation system. Guelat et al. [9] have presented a Gauss-Seidel Linear Approximation (GSLA) algorithm implemented for a multi-commodity, multi-mode nonlinear assignment problem.

In regards to integrality constraints on transportation flows, Li et al.[12] has given a DP algorithm for a single facility problem where incoming orders are multiple of a particular batch-size and the objective is to minimize cost of inventory, backlogs and production. Here the production cost is analogous to transportation cost in our problem. The major differences for the problem they address to ours is that in our problem there are multiple facilities, lot-sizes on various modes and we do not allow for indefinite backlogs.

Rabbani et al. [14] provides a solution to Supply Chain Network Design (SCND) planning for a multi-echelon network using an MIP model and a graph-theoretic heuristic. Rabbani et al. [13] presents MIP model for a multi-modal transportation problem in waste collection and a genetic algorithm (GA) for the same. Hanafi et al. [11] provides a solution for food supply chain industry using MIP model.

## 3 MILP model

### 3.1 Problem statement

We consider the following operational problem in logistics for a multi-echelon supply chain over multiple time periods and model it as a MILP. A space network of stock keeping facilities such as factories, warehouses, distribution centers, retailers, etc. is given. Goods produced in the supply chain must be transported from the upstream facilities to those downstream in the chain. Transportation decisions in such networks generally are: (a) How much quantity should be sent from one facility to the other (b) and when should this transportation take place. By finding good solutions to these questions, one can save costs of transporting goods and also of storing inventory. One would ideally like to supply goods from the nearest upstream facility to a given downstream facility. However, the required goods may not be available at the desired time or the upstream facility may have limited capacity, and so one may have to source the goods from other distant facilities at a higher cost. One can potentially store quantities of material to overcome shortages in later time periods, but storing inventory also incurs cost. By carefully planning the flow of goods, we can minimize the sum total of these costs.

Exogenous demand for goods, either deterministic or uncertain, must be met on time. The supply chain has to bear the cost if the demand in not met (lost sales) or met later than expected (backlogged). We assume an exogenous demand can be backlogged until a certain predefined time-period (maximum lateness) after which the demand is considered as a lost sale. Cost of lost sales and back-logging are also included in the model objective. Supply constraints exist on supply facilities which states that a finite supply is available at every time-period on a supply facility.

Sometimes, it is possible to transport goods through more than one mode of transportation. A mode may be cheaper and may transport large quantities in bulk, but it may also be slower than other modes. Often the transportation modes between two facilities have integer lot sizing constraints - i.e. if the quantity of flow is non-zero, then it must be an integer multiple of a specified quantity(lot size). Additional business rules may specify that some proportionality constraints be met on the choice of transport mode on a given facility, for example, a particular facility may specify that it sources a certain percentage of goods via each mode in every time period. Similarly, a facility may have constraints which require it to source certain quantity of materials through specific upstream suppliers only. These constraints are part of our model.

Discrete time horizons are considered over which decisions are to be made. It decides how much quantity must be transported over each mode between all pairs of facilities over the planning horizon. We observe that the model becomes large and intractable as the number of time periods increase. In order to overcome this difficulty, we adjust the granularity of the details of our model. The farther the time period from the time of modeling, more is the uncertainty in the demand and cost parameters of the model. Hence, we assume the plan for the first $N$ periods should be more detailed than the rest of time-horizon. We achieve this simplification by relaxing the lot-sizing restrictions of periods beyond $N$.

### 3.2 Assumptions

The following assumptions are made before we model the problem.

1. The exogenous demands and supply at facilities are known for the entire time-horizon.
2. Inventory holding and backlog costs are considered linear at each facility.
3. Backlogs are allowed upto maximum lateness ($S_i$) at facility $i$. After $S_i$ time-periods have elapsed, the unsatisfied demand becomes lost sales.
4. Backlog costs are provided as inputs. Demands not satisfied within maximum lateness period are considered as lost sales. Hence lost sales cost depends on maximum lateness and backlog costs.
5. There is only one commodity that is being transported or stored.
6. Perishability of the item is not considered. It is permitted to carry an item in inventory till the end of the time-horizon.
7. If material arrives at a facility at time $t$ via mode $m$, it can be made available for departure at time $t$ via mode $m'$; i.e. the transfer of material at a facility from one mode to another can occur on the same day.

### 3.3 Model

To formulate the model, we first introduce the following notation:

***Inputs and sets:***

$\mathscr{N}$ = set of facilities = $\{1,..,|\mathscr{N}|\}$
$\mathscr{M}$ = set of transportation modes = $\{1,..,|\mathscr{M}|\}$
$\mathscr{T}$ = set of time-periods = $\{1,..,|\mathscr{T}|\}$
$\mathscr{K}$ = set of time-buckets = $\{1,..,|\mathscr{K}|\}$
$\mathscr{T}_k = k^{th}$ bucket of time-periods $\subseteq \{1,..,|\mathscr{T}|\} \qquad k \in \mathscr{K}$
$\bigcup_{k \in \mathscr{K}} \mathscr{T}_k = \mathscr{T}$
$\bigcap_{k \in \mathscr{K}} \mathscr{T}_k = \varnothing$
$\delta^+(i)$ = set of facilities for which facility $i$ is a supplier
$\delta^-(i)$ = set of facilities for which facility $i$ is a customer
$S_i$ = maximum lateness allowable at facility $i$.
$H_{i,t}$ = inventory holding cost per unit item at facility $i$ to be carried from $t$ to $t+1$
$B_{i,t}$ = backlog cost per unit item at facility $i$ to be carried from $t+1$ to $t$
$LSC_{i,t}$ = lost sales cost per unit item at facility $i$ at time $t$
$C^m_{i,j}$ = transportation cost per unit item per unit time from facility $i$ to facility $j$ using mode $m$
$D_{i,t}$ = demand/supply at facility $i$ at time $t$
$L^m_{i,j}$ = time required for transportation from facility $i$ to facility $j$ using mode $m$
$A^m_{i,j}$ = lot size whose integer multiples are to be transported from facility $i$ to facility $j$ using mode $m$
$Q^m_{max,i}$ = upper limit on proportion of transportation flow to enter facility $i$ via mode $m$ over a time-bucket
$Q^m_{min,i}$ = lower limit on proportion of transportation flow to enter facility $i$ via mode $m$ over a time-bucket

$R^j_{max,i}$ = upper limit on proportion of transportaion flow to enter facility $i$ from facility $j$ over a time bucket
$R^j_{min,i}$ = lower limit on proportion of transportaion flow to enter facility $i$ from facility $j$ over a time bucket
$N$ = number of initial critical days
$E_{max}$ = Maximum of $\{S_i : i \in \mathcal{N}\} \cup \{L^m_{i,j} : m \in \mathcal{M}, i \in \mathcal{N}, j \in \delta^+(i)\}$

***Decision Variables:***

$x_{i,t}$ = inventory at facility $i$ that is carried from time $t$ to $t+1$
$y_{i,t}$ = backlog at facility $i$ that is carried from time $t+1$ to $t$
$f^m_{i,j,t}$ = integer multiplier for transportation from facility $i$ to $j$ using mode $m$ reaching at time $t$
$ls_{i,t}$ = lost sales occurring at facility $i$ at time $t$

***Formulation:***
***Objective:***

$$\text{minimize} \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} H_{i,t} \times x_{i,t} + \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} B_{i,t} \times y_{i,t} + \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} LSC_{i,t} \times ls_{i,t} + \sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{N}}\sum_{j\in\delta^+(i)}\sum_{t\in\mathcal{T}\cup\{|\mathcal{T}|+1,..,|\mathcal{T}|+E_{max}\}} L^m_{i,j} \times C^m_{i,j} \times A^m_{i,j} \times f^m_{i,j,t} \tag{OB}$$

***Constraints:*** *Flow balance constraints*

$$y_{i,1} + ls_{i,1} = x_{i,1} + D_{i,1} + \sum_{m\in\mathcal{M}}\sum_{j\in\delta^+(i)} A^m_{i,j} \times f^m_{i,j,1+L^m_{i,j}} \forall i \in \mathcal{N} \tag{FB-I}$$

$$x_{i,t-1} + \sum_{m\in\mathcal{M}}\sum_{j\in\delta^-(i)} A^m_{j,i} \times f^m_{j,i,t} + y_{i,t} + ls_{i,t} = x_{i,t} + \sum_{m\in\mathcal{M}}\sum_{j\in\delta^+(i)} A^m_{i,j} \times f^m_{i,j,t+L^m_{i,j}} + y_{i,t-1} + D_{i,t} \quad \forall i \in \mathcal{N}, t \in \{2,..,|\mathcal{T}|\} \tag{FB-II}$$

*Integrality and non-negativity constraints*

$$f^m_{i,j,t} \in \mathbb{Z}^+ \forall i \in \mathcal{N}, j \in \delta^+(i), t \in \{1,..,N\}, m \in \mathcal{M} \tag{INT}$$

$$f^m_{i,j,t} \in \mathbb{R}^+ \forall i \in \mathcal{N}, j \in \delta^+(i), t \in \{N+1,..,|\mathcal{T}|+E_{max}\}, m \in \mathcal{M} \quad x_{i,t}, y_{i,t}, ls_{i,t} \in \mathbb{R}^+ \forall i \in \mathcal{N}, t \in \mathcal{T} \tag{CONT}$$

*Fixing variables*

$$f^m_{i,j,t} = 0 \forall i \in \mathcal{N}, j \in \delta^+(i), m \in \mathcal{M}, t \in \{1,..,L^m_{i,j}\} \tag{FIX-f}$$

$$y_{i,t} = 0 \forall i \in \mathcal{N}, t \in \mathcal{T} \text{if} D_{i,t} \leq 0 \tag{FIX-y}$$

$$ls_{i,t} = 0 \forall i \in \mathcal{N}, t \in \mathcal{T} \text{if} D_{i,t} \leq 0 \tag{FIX-ls}$$

In the above model, the objective OB is to minimize the sum of inventory holding costs, backlog costs, lost sales costs and transportation costs in the network. To ensure that the backlog at facility $i (\forall i \in \mathcal{N})$ at any time $t (\forall t \in \mathcal{T})$ is not carried for more than maximum lateness $S_i$, we input lost sales costs such that $S_i \times B_{i,t} < LSC_{i,t} < (S_i + 1) \times B_{i,t}$. The constraint FB-I and FB-II are the flow balancing constraints on facilities at first time-period and consecutive time-periods respectively. Constraints INT and CONT are integrality and non-negativity constraints. In constraint FIX-f, we fix the transportation flows reaching before time $L^m_{i,j}$ at any facility $j$ from it's source $i$ via a mode $m$. The demand at facility $i$ at time $t$ $(D_{i,t})$ is used to represent exogenous supply as well as demand. For a supply facility $D_{i,t}$ will be a negative value and for a demand facility it will be a positive value as can be inferred from the flow constraints FB-I and FB-II. It can be noted that assumption 7 is built into constraints FB-I and FB-II as incoming and outgoing flows can be via different modes. If a facility $i$ is a pure supply facility or a facility where no exogenous demand$(D_{i,t})$ occurs at any time-period $t$, then there cannot be any backlogs or lost sales possible at $i$ as these backlogs and lost sales can be utilized only to satisfy exogenous demands. This is given by constraint FIX-y and FIX-ls. We refer to above model as ***FMIP***.

In ***FMIP*** (if solved directly using a solver), at every node the flow constraints are always satisfied because incoming flows additional to demand can be carried as inventory and if incoming flows are less then exogenous demand then that can be balanced by backlogs or lost sales. Moreover there are no negative cost cycles. Hence, ***FMIP*** will always have at least one feasible solution.

### 3.4 Business rule pertaining to modes

$$Q^m_{min,i}\left(\sum_{m' \in \mathcal{M}} \sum_{j \in \delta^-(i)} \sum_{t \in \mathcal{T}_k} A^{m'}_{j,i} \times f^{m'}_{j,i,t}\right) \leq \sum_{j \in \delta^-(i)} \sum_{t \in \mathcal{T}_k} A^m_{j,i} \times f^m_{j,i,t} \quad \forall k \in \mathcal{K}, i \in \mathcal{N}, m \in \mathcal{M} \tag{BR-1a}$$

$$\sum_{j \in \delta^-(i)} \sum_{t \in \mathcal{T}_k} A^m_{j,i} \times f^m_{j,i,t} \leq Q^m_{max,i}\left(\sum_{m' \in \mathcal{M}} \sum_{j \in \delta^-(i)} \sum_{t \in \mathcal{T}_k} A^{m'}_{j,i} \times f^{m'}_{j,i,t}\right) \quad \forall k \in \mathcal{K}, i \in \mathcal{N}, m \in \mathcal{M} \tag{BR-1b}$$

The business proportionality constraints pertaining to modes can be expressed as BR-1a and BR-1b. It states that sum of incoming transportation flows to facility $i$ from all of its suppliers summed over time-periods in a given time-bucket via a specific mode $m$ is bounded by lower and upper limit on proportion of sum of flows over all suppliers, time-periods in a given time-bucket and via all modes. When BR-1a and BR-1b are added as constraints to ***FMIP***, we refer to the model as ***MIP-BR1***.

### 3.5 Business rule pertaining to sources

$$R^{j}_{min,i}\left(\sum_{m\in\mathcal{M}}\sum_{j'\in\delta^{-}(i)}\sum_{t\in\mathcal{T}_k} A^{m}_{j',i}\times f^{m}_{j',i,t}\right)\leq\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}_k} A^{m}_{j,i}\times f^{m}_{j,i,t} \quad \forall k\in\mathcal{K}, i\in\mathcal{N}, j\in\delta^{-}(i) \tag{BR-2a}$$

$$\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}_k} A^{m}_{j,i}\times f^{m}_{j,i,t}\leq R^{j}_{max,i}\left(\sum_{m\in\mathcal{M}}\sum_{j'\in\delta^{-}(i)}\sum_{t\in\mathcal{T}_k} A^{m}_{j',i}\times f^{m}_{j',i,t}\right) \quad \forall k\in\mathcal{K}, i\in\mathcal{N}, j\in\delta^{-}(i) \tag{BR-2b}$$

The business proportionality constraints pertaining to sources can be expressed as BR-2a and BR-2b. It states that sum of incoming transportation flows to facility $i$ from all modes and summed over time-periods in a given time-bucket from a specific supplier $j$ is bounded by lower and upper limit on proportion of sum of flows from all modes, suppliers and time-periods in a given time-bucket. When BR-2a and BR-2b are added as constraints to ***FMIP***, we refer to the model as ***MIP-BR2***.

When BR-1a, BR-1b, BR-2a and BR-2b are added as constraints to ***FMIP***, we refer to the model as ***MIP-BR12***.

## 4 Network representation

The problem discussed can be presented as time-space network $G(\mathcal{N}\times\mathcal{T},\mathcal{A})$ where $\mathcal{N}\times\mathcal{T}$ represents the set of nodes in the network and $\mathcal{A}$ represents set of arcs connecting these nodes. The flows on arcs between the nodes are either transportation, inventory or backlogs. There are exogenous flows for supply, demand and lost sales. Figure 1 shows the various flows incoming and outgoing a typical node in the network.

A node in the graph denotes the pair $(i, t)$, where $i$ is a facility in the supply chain and $t$ is the discrete time-period (e.g. *a day*). The node $(i, t)$ is connected to node $(k, t + L^{m}_{ik})$ where the facility $i$ is connected to $k$ with an arc taking transportation time $L^{m}_{ik}$ via mode $m$.

Figure 2 shows a snapshot of an example of $G(\mathcal{N}\times\mathcal{T},\mathcal{A})$ where there are four facilities and four time-periods for representation purpose.

## 5 Rounding heuristic

In this section we propose heuristics to get feasible solution to the problem. We make an attempt to get a feasible solution to MILPs: ***FMIP***, ***MIP-BR1*** , ***MIP-BR2*** and ***MIP-BR12***.

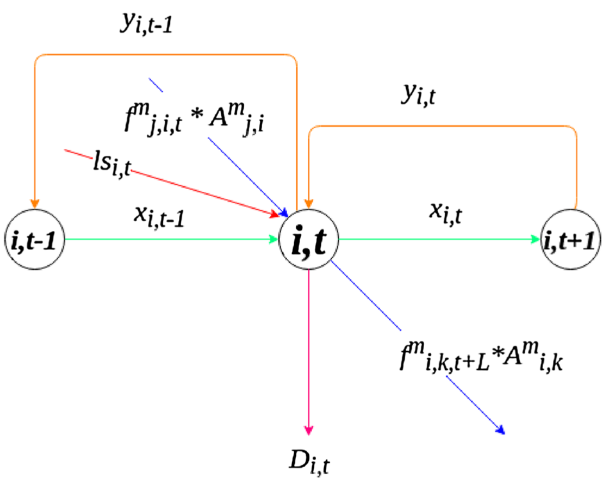


**Fig. 1** Flows through a node in network

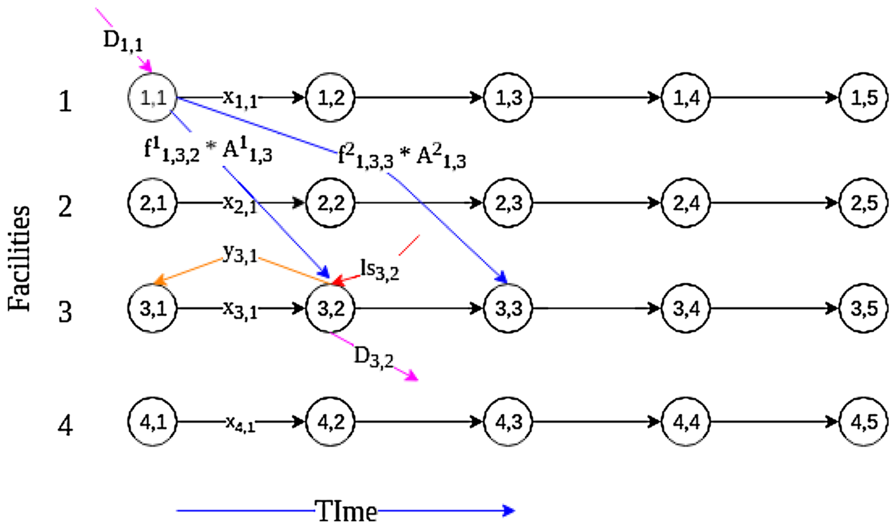


**Fig. 2** Time-space network representation of an instance of the problem

### 5.1 Rounding heuristic for *FMIP*

In this section we present two rounding heuristics to get a feasible solution to ***FMIP***. We solve a sequence of LPs to get to an integer feasible solution of ***FMIP***. First, we relax the constraints INT in ***FMIP***. This gives us an initial LP model of network flow problem with flow balance constraints.

### 5.1.1 Heuristic-I

In this heuristic we begin by solving the LP relaxation of ***FMIP*** and set $\tau \leftarrow 1 + \min(L^m_{i,j})$ because at this time-period first non-zero incoming transportation flow may occur at any facility. The solution of this LP is set as $(\hat{x}.\hat{y}, \hat{f}, \hat{ls})_\tau$ where $\tau = 1 + \min(L^m_{i,j})$. In this solution we look at all transportation flows $f^m_{i,j,\tau}$ i.e. flows reaching to facility $j$ from facility $i$ at time-period $\tau$ via mode $m$ since the earliest delivery of transportation flow occurs at this time-period. For each of these transportation flows (variables) in LP solution, we round-up the decision value if there is sufficient inventory at source node of the transportation flow, otherwise we round-down. After this fixing and adding them as constraints, we solve the modified LP again and increment $\tau$ by 1. This process is repeated in iteration till $\tau = N$. Heuristic-I is shown in Algorithm 1.

In an iteration $\tau$, the values in $(\hat{f})_\tau$ (solution before rounding) are either fractional or integer. If the values in $(\hat{f})_\tau$ are rounded up given there is sufficient inventory at the source node, we do not incur any additional backlogs at the target node and demands are satisfied better than previous solution at the target node. But if there is not sufficient inventory at the source node, then we have to round down the values in $(\hat{f})_\tau$ as rounding up will violate the flow constraints FB-I and FB-II at the source node.

**Algorithm 1** Heuristic-I

**Input:** All parameters related to the problem
**Output:** Feasible solution to the **FMIP**

Formulate **FMIP** from the inputs
Solve LP-relaxation of **FMIP**
$\tau \leftarrow 1 + \min(L^m_{i,j})$
Set the solution of LP-relaxation to $(\hat{x}.\hat{y}, \hat{f}, \hat{ls})$
**while** $\tau \leq N$ **do**
  **for** $m \in \mathscr{M}$ **do**
    **for** $i \in \mathscr{N}$ **do**
      **for** $j \in \delta^-(i)$ **do**
        **if** $\hat{x}_{j,\tau-L^m_{j,i}} \geq (\lceil \hat{f}^m_{j,i,\tau} \rceil - \hat{f}^m_{j,i,\tau}) \times A^m_{j,i}$ **then**
          Add constraint: $f^m_{j,i,\tau} = \lceil \hat{f}^m_{j,i,\tau} \rceil$ to **FMIP**
          $\hat{x}_{j,\tau-L^m_{j,i}} \leftarrow \hat{x}_{j,\tau-L^m_{j,i}} - (\lceil \hat{f}^m_{j,i,\tau} \rceil - \hat{f}^m_{j,i,\tau}) \times A^m_{j,i}$
        **else**
          Add constraint: $f^m_{j,i,\tau} = \lfloor \hat{f}^m_{j,i,\tau} \rfloor$ to **FMIP**
          $\hat{x}_{j,\tau-L^m_{j,i}} \leftarrow \hat{x}_{j,\tau-L^m_{j,i}} + (\hat{f}^m_{j,i,\tau} - \lfloor \hat{f}^m_{j,i,\tau} \rfloor) \times A^m_{j,i}$
        **end if**
      **end for**
    **end for**
  **end for**
  Solve LP-relaxation of updated **FMIP**
  $\tau \leftarrow \tau + 1$
  Update solution $(\hat{x}.\hat{y}, \hat{f}, \hat{ls})$
**end while**

### 5.1.2 Heuristic-II

In this heuristic we begin by solving the LP relaxation of ***FMIP*** and set $\tau \leftarrow 1$ because at this time-period first non-zero outgoing transportation flow may occur from any facility. In the solution of this LP, $(\hat{x}.\hat{y},\hat{f},\hat{ls})_\tau$ where $\tau = 1$, we look at all transportation flows $f^m_{i,j,\tau+L^m_{i,j}}$ i.e. flows reaching to facility $j$ from facility $i$ at time-period $\tau + L^m_{i,j}$ via mode $m$. For each of these transportation flows (variables) in LP solution, we round-up the decision value if there is sufficient inventory at source of the transportation flow, otherwise we round-down. After this fixing of variables and adding them as constraints, we solve the modified LP again and increment $\tau$ by 1. This process is repeated in iteration till $\tau = N - \min(L^m_{i,j})$. Heuristic-II is shown in Algorithm 2.

The conditions required to be satisfied for rounding up/down of $(\hat{f})_\tau$ are in steps (1.9,2.9).

**Algorithm 2** Heuristic-II

**Input:** All parameters related to the problem
**Output:** Feasible solution to the **FMIP**

1: Formulate **FMIP** from the inputs
2: Solve LP-relaxation of **FMIP**
3: $\tau \leftarrow 1$
4: Set the solution of LP-relaxation to $(\hat{x}.\hat{y},\hat{f},\hat{ls})$
5: **while** $\tau \leq N - \min(L^m_{i,j})$ **do**
6:   **for** $m \in \mathscr{M}$ **do**
7:     **for** $i \in \mathscr{N}$ **do**
8:       **for** $j \in \delta^+(i)$ **do**
9:         **if** $\hat{x}_{i,\tau} \geq (\lceil \hat{f}^m_{i,j,\tau+L^m_{i,j}} \rceil - \hat{f}^m_{i,j,\tau+L^m_{i,j}}) \times A^m_{i,j}$ **then**
10:           Add constraint: $f^m_{i,j,\tau+L^m_{i,j}} = \lceil \hat{f}^m_{i,j,\tau+L^m_{i,j}} \rceil$ to **FMIP**
11:           $\hat{x}_{i,\tau} \leftarrow \hat{x}_{i,\tau} - (\lceil \hat{f}^m_{i,j,\tau+L^m_{i,j}} \rceil - \hat{f}^m_{i,j,\tau+L^m_{i,j}}) \times A^m_{i,j}$
12:         **else**
13:           Add constraint: $f^m_{i,j,\tau+L^m_{i,j}} = \lfloor \hat{f}^m_{i,j,\tau+L^m_{i,j}} \rfloor$ to **FMIP**
14:           $\hat{x}_{i,\tau} \leftarrow \hat{x}_{i,\tau} + (\hat{f}^m_{i,j,\tau+L^m_{i,j}} - \lfloor \hat{f}^m_{i,j,\tau+L^m_{i,j}} \rfloor) \times A^m_{i,j}$
15:         **end if**
16:       **end for**
17:     **end for**
18:   **end for**
19:   Solve LP-relaxation of updated **FMIP**
20:   $\tau \leftarrow \tau + 1$
21:   Update solution $(\hat{x}.\hat{y},\hat{f},\hat{ls})$
22: **end while**

## 5.2 Rounding heuristic for *MIP-BR1*, *MIP-BR2* and *MIP-BR12*

$$f^m_{i,j,\max(\mathscr{T}_k)} \in \mathbb{Z}^+ \forall i \in \mathscr{N}, j \in \delta^+(i), m \in \mathscr{M}, k \in \mathscr{K} \text{and} \max(\mathscr{T}_k) \leq N \tag{INT-RH}$$

Initially we relax the constraints INT in ***MIP-BR1***, ***MIP-BR2*** and ***MIP-BR12*** to INT-RH We refer to these MILPs as ***RMIP-BR1***, ***RMIP-BR2*** and ***RMIP-BR12***. The heuristics proposed here are quite similar to Algorithm 1 and Algorithm 2. The difference here is instead of solving a sequence of LPs, we are solving a sequence of MILPs with reduced number of integer variables.

The decision to keep variables given in INT-RH as integers is because once the $f$'s corresponding to time-periods $\{\theta_1, \theta_2, \theta_3, .., \theta_{|\mathscr{T}_k|-1}\} \subset \mathscr{T}_k$ of a time-bucket $k$ are fixed as integers, the rounding step may not give a feasible solution because there are no future time-periods in that time-bucket where these flows can be adjusted by just solving the LP relaxation of the problem. Here our business rules restrict us from not to proceed with rounding step, hence these variables are declared as integer variables in the problem.

### 5.2.1 Heuristic-III

In this heuristic we begin by solving ***RMIP-BRx*** and set $\tau \leftarrow 1 + \min(L^m_{i,j})$ because at this time-period first non-zero incoming transportation flow may occur at any facility. In the solution of the initial MILP $(\hat{x}.\hat{y}, \hat{f}, \hat{ls})_\tau$ where $\tau = 1 + min(L^m_{i,j})$, we look at all transportation flows $f^m_{i,j,\tau}$ i.e. flows reaching to facility $j$ at time-period $\tau$ from facility $i$ via mode $m$ since the earliest delivery of transportation flow occurs at this time-period. For each of these variables in MILP solution, we round-up the decision value of these transportation flows (variables) if there is sufficient inventory at source of the transportation flow, otherwise we round-down. After this fixing and adding them as constraints, we solve the modified MILP and increment $\tau$ by 1. This process is repeated in iteration till $\tau = N$. Heuristic-III is shown in Algorithm 3.

**Algorithm 3** Heuristic-III

**Input:** Input parameters, sets and business rule to be followed
**Output:** Feasible solution to the **MIP-BRx**

Formulate ***RMIP-BRx*** from the input and as per the business rule-x to be followed
Solve ***RMIP-BRx*** on MILP-solver with a time-limit and fractional gap
$\tau \leftarrow 1 + min(L^m_{i,j})$
Set the solution of ***RMIP-BRx*** to $(\hat{x}.\hat{y}, \hat{f}, \hat{ls})_\tau$
**while** $\tau \le N$ **do**
  **for** $k \in \mathscr{K}$ **do**
    **if** $\tau \in \mathscr{T}_k$ **then**
      $TB \leftarrow \mathscr{T}_k$
    **end if**
  **end for**
  **if** $\tau \neq \max(TB)$ **then**
    **for** $m \in \mathscr{M}$ **do**
      **for** $i \in \mathscr{N}$ **do**
        **for** $j \in \delta^-(i)$ **do**
          **if** $\hat{x}_{j,\tau-L^m_{j,i}} \ge (\lceil \hat{f}^m_{j,i,\tau} \rceil - \hat{f}^m_{j,i,\tau}) \times A^m_{j,i}$ **then**
            Add constraint $f^m_{j,i,\tau} = \lceil \hat{f}^m_{j,i,\tau} \rceil$ to ***RMIP-BRx***
            $\hat{x}_{j,\tau-L^m_{j,i}} \leftarrow \hat{x}_{j,\tau-L^m_{j,i}} - (\lceil \hat{f}^m_{j,i,\tau} \rceil - \hat{f}^m_{j,i,\tau}) \times A^m_{j,i}$
          **else**
            Add constraint $f^m_{j,i,\tau} = \lfloor \hat{f}^m_{j,i,\tau} \rfloor$ to ***RMIP-BRx***
            $\hat{x}_{j,\tau-L^m_{j,i}} \leftarrow \hat{x}_{j,\tau-L^m_{j,i}} + (\hat{f}^m_{j,i,I} - \lfloor \hat{f}^m_{j,i,\tau} \rfloor) \times A^m_{j,i}$
          **end if**
        **end for**
      **end for**
    **end for**
    Solve updated ***RMIP-BRx***
    $\tau \leftarrow \tau + 1$
    Update solution to $(\hat{x}.\hat{y}, \hat{f}, \hat{ls})_\tau$
  **else**
    $\tau \leftarrow \tau + 1$
  **end if**
**end while**

### 5.2.2 Heuristic-IV

In this heuristic we begin by solving ***RMIP-BRx*** and set $\tau \leftarrow 1$ because at this time-period first non-zero outgoing transportation flow may occur from any facility. In the solution of the initial MILP, $(\hat{x}.\hat{y},\hat{f},\hat{ls})_\tau$ where $\tau = 1$, we look at all transportation flows $f^m_{i,j,\tau+L^m_{i,j}}$ i.e. flows reaching facility $j$ from facility $i$ at time-period $\tau + L^m_{i,j}$ using mode $m$. For each of these variables in MILP solution, we round-up the decision value if there is sufficient inventory at source of the transportation flow, otherwise we round-down. After this fixing and adding them as constraints, we solve the modified MILP again and increment $\tau$ by 1. This process is repeated in iteration till $\tau = N - \min(L^m_{i,j})$. This is shown in Algorithm 4.

**Algorithm 4** Heuristic-IV

**Input:** All parameters related to the problem
**Output:** Feasible solution to the **MIP-BRx**

Formulate ***RMIP-BRx*** from the input and as per the business rule-x to be followed
Solve ***RMIP-BRx*** on MILP-solver with a time-limit and fractional gap
$\tau \leftarrow 1$
Set the solution of ***RMIP-BRx*** to $(\hat{x}.\hat{y},\hat{f},\hat{ls})_\tau$
**while** $\tau \leq N - \min(L^m_{i,j})$ **do**
  **for** $k \in \mathscr{K}$ **do**
    **if** $\tau \in \mathscr{T}_k$ **then**
      $TB \leftarrow \mathscr{T}_k$
    **end if**
  **end for**
  **if** $\tau \neq \max(TB)$ **then**
    **for** $m \in \mathscr{M}$ **do**
      **for** $i \in \mathscr{N}$ **do**
        **for** $j \in \delta^+(i)$ **do**
          **if** $\hat{x}_{i,\tau} \geq (\lceil \hat{f}^m_{i,j,\tau+L^m_{i,j}} \rceil - \hat{f}^m_{i,j,\tau+L^m_{i,j}}) \times A^m_{i,j}$ **then**
            Add constraint: $f^m_{i,j,\tau+L^m_{i,j}} = \lceil \hat{f}^m_{i,j,\tau+L^m_{i,j}} \rceil$ to **RMIP-BRx**
            $\hat{x}_{i,\tau} \leftarrow \hat{x}_{i,\tau} - (\lceil \hat{f}^m_{i,j,\tau+L^m_{i,j}} \rceil - \hat{f}^m_{i,j,\tau+L^m_{i,j}}) \times A^m_{i,j}$
          **else**
            Add constraint: $f^m_{i,j,\tau+L^m_{i,j}} = \lfloor \hat{f}^m_{i,j,\tau+L^m_{i,j}} \rfloor$ to **RMIP-BRx**
            $\hat{x}_{i,\tau} \leftarrow \hat{x}_{i,\tau} + (\hat{f}^m_{i,j,\tau+L^m_{i,j}} - \lfloor \hat{f}^m_{i,j,\tau+L^m_{i,j}} \rfloor) \times A^m_{i,j}$
          **end if**
        **end for**
      **end for**
    **end for**
    Solve LP-relaxation of updated **FMIP**
    $\tau \leftarrow \tau + 1$
    Update solution $(\hat{x}.\hat{y},\hat{f},\hat{ls})$
  **else**
    $\tau \leftarrow \tau + 1$
  **end if**
**end while**

## 5.3 Impact of rounding heuristic

In rounding heuristics Algorithm 3 and Algorithm 4, the number of integer decision variables got reduced from $\mathcal{O}(|\mathscr{M}| \times |\mathscr{A}_s| \times N)$ in ***MIP-BR1***, ***MIP-BR2*** and ***MIP-BR12*** to $\mathcal{O}(|\mathscr{M}| \times |\mathscr{A}_s| \times k')$ where $k'$ refers to index of the last time-bucket such that $max(\mathscr{T}_{k'}) \leq N$ in ***RMIP-BR1***, ***RMIP-BR2*** and ***RMIP-BR12***. The effect on objective when $(\hat{f})_\tau$ are rounded up and added as constraint to iteration $\tau + 1$ can be given as in equation 1. The effect on objective when fixing $f$'s to floor of previous iteration values is difficult to quantify as the reduced flow can be compensated either by flows from other modes which could have been set to ceiling, backlogs, or lost sales.

$$\begin{aligned}\text{Cost}_{\tau+1} =&\text{Cost}_{\tau} + (\lceil \hat{f}^m_{j,i,\tau} \rceil - \hat{f}^m_{j,i,\tau})_\tau \times A^m_{j,i} \times C^m_{j,i} \times L^m_{j,i} \\ &- [\hat{x}_{j,\tau-L^m_{j,i}} - (\lceil \hat{f}^m_{j,i,\tau} \rceil - \hat{f}^m_{j,i,\tau}) \times A^m_{j,i}]_\tau \times H_{j,\tau-L^m_{j,i}}\end{aligned} \tag{1}$$

## 6 Computational results

We discussed a MILP model in Sect. 3 to address the problem statement and heuristics to get a feasible solution. In this section we present our experimental results on data-sets for model solving and various heuristics discussed in sects. 3 and 5 using CPLEX-12.6.1.0 on Intel(R) Xeon(R) CPU E5-2670 v2 2.50GHz with 128 GB memory. Data for instances(ds1,ds2,ds3,net_V,net_X) such as demands, supply, costs and lead-times considered were randomly generated for various number of facilities, modes and time-periods. Table 1 provides high-level properties of instances used in experiments. The echelon gives the number of nodes in a supply-chain echelon (e.g. echelon 1 can be considered as factories, echelon 2 as distribution centers/warehouses and so on). Section 6.1 gives the analysis of these results. The following approaches have been considered to solve the instances:

1. The problem modeled as an MILP is solved on CPLEX-12.6.1.0 with time-limit as 1800 seconds and 'mipgap' set as 0.01. **T_O_CPLEX**
2. The problem is solved using heuristic (Heuristic-I for ***FMIP*** and Heuristic-III for ***MIP-BRx***). **T_H_CPLEX_I** or **T_H_CPLEX_III**
3. The problem is solved using heuristic (Heuristic-II for ***FMIP*** and Heuristic-IV for ***MIP-BRx***). **T_H_CPLEX_II** or **T_H_CPLEX_IV**
4. The solution provided by one of the heuristic is used as 'MIP start' in CPLEX-12.6.1.0 for solving the model with time-limit set as 1800 seconds and 'mipgap' as 0.01. **T_OI_CPLEX**

**Table 1** Data-set properties

| | ds1_a | ds2_a | ds3_a | net_V | net_X |
|---|---|---|---|---|---|
| Echelon 1 | 4 | 4 | 4 | 1 | 2 |
| Echelon 2 | 4 | 4 | 4 | 2 | 3 |
| Echelon 3 | 30 | 30 | 20 | 2 | 5 |
| Echelon 4 | 0 | 0 | 151 | 0 | 0 |
| $\lvert\mathcal{N}\rvert$ | 38 | 38 | 179 | 5 | 10 |
| $\lvert\mathcal{T}\rvert$ | 61 | 61 | 61 | 50 | 50 |
| $\lvert\mathcal{M}\rvert$ | 2 | 2 | 2 | 2 | 2 |
| $\lvert\mathcal{K}\rvert$ | 2 | 2 | 2 | 5 | 5 |
| N | (30,40,50) | (30,40,50) | (30,40,50) | (10,20,30,40) | (10,20,30,40) |

In implementation of Heuristic-III (**T_H_CPLEX_III**) and Heuristic-IV (**T_H_CPLEX_IV**), for each iteration time-limit is set as 1000 seconds and 'mipgap' as 0.01.

We represent analysis using the time required in each approach as well as the gap of solution found from best known bound if that method was independently used.

### 6.1 Analysis of results

Algorithm 1 to Algorithm 4 do not always find a feasible solution to the problem as shown by the following example 'net_X'.

While solving ***FMIP*** model for net_X instance using Algorithm 1 at iteration $\tau = 4$ obtains in its solution $x_{4,2} = 6$. When we go to next iteration $\tau = 5$, we consume 5 units from $x_{4,2}$ to round up a value a transportation flow reaching at some facility at $\tau = 5$ thus making $x_{4,2} = 1$. But before this step happens, one of the flows reaching at some other facility at time $\tau = 5$ is round down and adds 4 units of inventory at $x_{4,3}$ making $x_{4,3} = 4$. Thus at location 4, we end up needing $x_{4,2} = 1$ and $x_{4,3} = 4$ in a situation where all other incoming and outgoing transportation flows are fixed. This leads to violation of flow-constraint at node (4,3) in this instance. Thus, we have a counter-example for Heuristic-I and Heuristic-III which follow similar approach for rounding that does not give a feasible solution.

Algorithm 2 overcomes this drawback of algorithm 1. Here we look at all outgoing flows at each iteration, hence the above issue is remedied as at an iteration there are no chances that any previous time period inventory flow will be modified.

As can be seen from Tables 2, 3, 4 and 5 one of the proposed algortihms always provided a feasible solution to instances in our experiments.

There cannot be any dominance relationship established between the Heuristic-III and Heuristic-IV as we have cases where Algorithm 3 gave a feasible solution but 4 doesn't and vice-versa. Based on our experiments we can say that Heuristic-II dominates Heuristic-I but we do not generalize this statement.

We have highlighted in **bold** the results where % gap for **T_O_CPLEX** differs from % gap for **T_OI_CPLEX** by more than 40%. This shows significance of heuristics proposed, as using heuristic solution as an MIP-start helped in achieving a solution within 1% gap from lower bound (LB) for 34 out of 68 instances within similar time-limits.

For 41 instances heuristics were able to give a solution within 3% gap from LP solution in less than 600 seconds. It is evident that for heuristics the % gap from LP solution is non-decreasing with increasing value of $N$ for given data-sets.

It can be noted that as value of initial critical days ($N$), number of facilities ($|\mathcal{M}|$) increases, for same solution time limit the gap from optimality increases for **T_O_CPLEX**, while applying the heuristics to get an initial feasible solution and utilizing it as an MIP-start in solver gives significant improvements in optimality gap in less time. As for other parameters, it is not possible to comment on the impact of change in them on the time required to obtain a solution.

**Table 2** FMIP results for instances

| | | | Rounding Heuristic I | | Rounding Heuristic II | | CPLEX Optimality(time limit 1800s) | | CPLEX Optimality with initial feasible from heuristic | |
|---|---|---|---|---|---|---|---|---|---|---|
| Instance | N | LP | T_H_CPLEX_I | %gap from LP | T_H_CPLEX_II | %gap from LP | T_O_CPLEX | %gap from LB | T_OI_CPLEX | %gap from LB |
| **ds1_a** | **30** | **2.00E+08** | **Infeasible** | | **177.07** | **0.07** | **1805.88** | **47.99** | **178.12** | **0.07** |
| **ds1_a** | **40** | **2.00E+08** | **Infeasible** | | **195.76** | **0.08** | **1804.64** | **58.19** | **196.08** | **0.08** |
| **ds1_a** | **50** | **2.00E+08** | **Infeasible** | | **180.34** | **0.10** | **1807** | **75.11** | **180.64** | **0.10** |
| ds2_a | 30 | 2.01E+08 | Infeasible | | 131.96 | 0.11 | 396.84 | 0.36 | 133.57 | 0.11 |
| ds2_a | 40 | 2.01E+08 | Infeasible | | 182.83 | 0.12 | 533.71 | 0.94 | 184.37 | 0.12 |
| ds2_a | 50 | 2.01E+08 | Infeasible | | 210.03 | 0.14 | 928.45 | 0.65 | 211.2 | 0.13 |
| **ds3_a** | **30** | **6.74E+08** | **Infeasible** | | **355.74** | **0.10** | **1802.2** | **61.12** | **360.03** | **0.09** |
| **ds3_a** | **40** | **6.74E+08** | **Infeasible** | | **449.96** | **0.19** | **1803.85** | **76.15** | **453.62** | **0.18** |
| **ds3_a** | **50** | **6.74E+08** | **Infeasible** | | **561.86** | **0.29** | **1804.48** | **82.78** | **565.59** | **0.28** |
| net_V | 10 | 271400 | 3.58 | 2.82 | 1.03 | 1.07 | 2.11 | 0.99 | 5.13 | 0.97 |
| net_V | 20 | 271400 | 1.75 | 5.78 | 1.95 | 1.93 | 24.09 | 1.00 | 26.82 | 0.96 |
| net_V | 30 | 271400 | 2.6 | 8.47 | 2.78 | 2.88 | 1801.34 | 1.35 | 43.66 | 0.98 |
| net_V | 40 | 271400 | 4.97 | 10.89 | 3.68 | 4.06 | 1802 | 1.57 | 1807.18 | 1.35 |
| net_X | 10 | 804630 | Infeasible | | 5.5 | 1.47 | 3.47 | 0.95 | 7.16 | 0.71 |
| net_X | 20 | 804630 | Infeasible | | 12.18 | 3.53 | 978.4 | 1.00 | 1814.51 | 1.07 |
| net_X | 30 | 804630 | Infeasible | | 13.78 | 5.06 | 961.78 | 0.93 | 1816.25 | 4.75 |
| net_X | 40 | 804630 | Infeasible | | 9.84 | 6.05 | 1802.11 | 1.99 | 1811.2 | 5.71 |

**Table 3** MIP-BR1 results for instances

| | | | Rounding Heuristic III | | Rounding Heuristic IV | | CPLEX Optimality(time limit 1800s) | | CPLEX Optimality with initial feasible from heuristic | |
|---|---|---|---|---|---|---|---|---|---|---|
| Instance | N | LP | T_H_CPLEX_III | %gap from LP | T_H_CPLEX_IV | %gap from LP | T_O_CPLEX | %gap from LB | T_OI_CPLEX | %gap from LB |
| ds1_a | 30 | 2.13E+08 | Infeasible | | 144.26 | 0.06 | 241.03 | 0.20 | 146.04 | 0.06 |
| **ds1_a** | **40** | **2.13E+08** | **Infeasible** | | **471.29** | **0.08** | **1,805.00** | **46.73** | **472.74** | **0.08** |
| **ds1_a** | **50** | **2.13E+08** | **Infeasible** | | **270.40** | **0.09** | **1,807.25** | **66.22** | **271.41** | **0.09** |
| ds2_a | 30 | 2.13E+08 | Infeasible | | 228.35 | 0.06 | 389.17 | 0.65 | 230.66 | 0.05 |
| **ds2_a** | **40** | **2.13E+08** | **Infeasible** | | **289.12** | **0.07** | **1,804.64** | **52.06** | **290.51** | **0.07** |
| **ds2_a** | **50** | **2.13E+08** | **Infeasible** | | **347.20** | **0.08** | **1,804.72** | **68.69** | **348.66** | **0.08** |
| **ds3_a** | **30** | **6.87E+08** | **Infeasible** | | **566.17** | **0.08** | **1,808.57** | **61.13** | **570.89** | **0.08** |
| **ds3_a** | **40** | **6.87E+08** | **Infeasible** | | **612.72** | **0.17** | **1,810.57** | **75.54** | **616.51** | **0.16** |
| **ds3_a** | **50** | **6.87E+08** | **Infeasible** | | **763.02** | **0.27** | **1,815.25** | **82.57** | **767.46** | **0.26** |
| net_V | 10 | 295722.5 | 7.62 | 1.85 | Infeasible | | 2.39 | 0.31 | 9.67 | 0.46 |
| net_V | 20 | 295722.5 | Infeasible | | 4.63 | 2.15 | 27.72 | 0.87 | 15.75 | 0.99 |
| net_V | 30 | 295722.5 | 16.3 | 10.42 | 5.35 | 2.79 | 1872.29 | 2.85 | 960.26 | 1.00 |
| net_V | 40 | 295722.5 | Infeasible | | 6.41 | 4.05 | 1801.79 | 1.47 | 1,808.85 | 2.37 |
| net_X | 10 | 1035974.46 | 11.81 | 1.27 | Infeasible | | 1.79 | 0.76 | 14.62 | 0.86 |
| net_X | 20 | 1035974.46 | 30.09 | 4.08 | Infeasible | | 1,801.44 | 1.95 | 1,834.74 | 1.89 |
| **net_X** | **30** | **1035974.46** | **11.1** | **7.34** | **Infeasible** | | **1,802.19** | **56.76** | **1,812.55** | **3.67** |
| **net_X** | **40** | **1035974.46** | **60.78** | **10.38** | **Infeasible** | | **1,802.23** | **72.94** | **1,862.90** | **5.85** |

**Table 4** MIP-BR2 results for instances

| | | | Rounding Heuristic III | | Rounding Heuristic IV | | CPLEX Optimality(time limit 1800s) | | CPLEX Optimality with initial feasible from heuristic | |
|---|---|---|---|---|---|---|---|---|---|---|
| Instance | N | LP | T_H_CPLEX_III | %gap from LP | T_H_CPLEX_IV | %gap from LP | T_O_CPLEX | %gap from LB | T_OI_CPLEX | %gap from LB |
| ds1_a | 30 | 2.01E+08 | Infeasible | | 190.00 | 0.08 | 161.00 | 0.71 | 191.65 | 0.08 |
| **ds1_a** | **40** | **2.01E+08** | **Infeasible** | | **241.64** | **0.09** | **1,805.31** | **66.26** | **243.04** | **0.09** |
| ds1_a | 50 | 2.01E+08 | Infeasible | | 300.58 | 0.11 | 1,805.00 | 1.39 | 301.80 | 0.10 |
| ds2_a | 30 | 2.01E+08 | Infeasible | | 218.11 | 0.08 | 312.87 | 0.66 | 220.14 | 0.08 |
| ds2_a | 40 | 2.01E+08 | Infeasible | | 283.27 | 0.09 | 1,805.74 | 34.10 | 284.68 | 0.09 |
| **ds2_a** | **50** | **2.01E+08** | **Infeasible** | | **348.46** | **0.10** | **1,809.78** | **68.73** | **349.70** | **0.10** |
| **ds3_a** | **30** | **6.89E+08** | **Infeasible** | | **511.40** | **0.08** | **1,818.66** | **61.18** | **516.61** | **0.07** |
| **ds3_a** | **40** | **6.89E+08** | **Infeasible** | | **658.17** | **0.17** | **1,821.74** | **75.74** | **663.01** | **0.16** |
| **ds3_a** | **50** | **6.89E+08** | **Infeasible** | | **797.08** | **0.28** | **1,812.30** | **82.39** | **801.24** | **0.28** |
| net_V | 10 | 291640 | 7.22 | 2.77 | 1.76 | 1.00 | 2.19 | 0.85 | 9.07 | 0.48 |
| net_V | 20 | 291640 | 3.07 | 7.50 | 3.06 | 2.50 | 5.09 | 0.99 | 12.74 | 0.95 |
| net_V | 30 | 291640 | 6.73 | 9.97 | 4.36 | 4.25 | 1,671.80 | 1.00 | 1,809.36 | 1.21 |
| net_V | 40 | 291640 | 9.64 | 11.14 | 5.56 | 4.80 | 1,802.63 | 2.70 | 1,811.82 | 3.45 |
| net_X | 10 | 992430 | 12.78 | 1.46 | 3.09 | 1.16 | 7.71 | 0.68 | 15.80 | 0.77 |
| net_X | 20 | 992430 | 28.87 | 4.58 | 5.63 | 2.76 | 1,801.34 | 1.25 | 1,831.81 | 4.43 |
| **net_X** | **30** | **992430** | **11.07** | **7.55** | **Infeasible** | | **1,802.45** | **59.58** | **1,812.45** | **7.31** |
| **net_X** | **40** | **992430** | **56.27** | **9.26** | **Infeasible** | | **1,801.63** | **83.08** | **1,858.39** | **8.98** |

**Table 5** MIP-BR12 results for intances

| | | | Rounding Heuristic III | | Rounding Heuristic IV | | CPLEX Optimality(time limit 1800s) | | CPLEX Optimality with initial feasible from heuristic | |
|---|---|---|---|---|---|---|---|---|---|---|
| Instance | N | LP | T_H_CPLEX_III | %gap from LP | T_H_CPLEX_IV | %gap from LP | T_O_CPLEX | %gap from LB | T_OI_CPLEX | %gap from LB |
| ds1_a | 30 | 2.13E+08 | Infeasible | | 197 | 0.07 | 239.71 | 0.26 | 199.44 | 0.06 |
| **ds1_a** | **40** | **2.13E+08** | **Infeasible** | | **270.55** | **0.08** | **1805** | **64.80** | **272.27** | **0.08** |
| **ds1_a** | **50** | **2.13E+08** | **Infeasible** | | **321** | **0.09** | **1806.72** | **69.95** | **322.77** | **0.09** |
| ds2_a | 30 | 2.13E+08 | Infeasible | | 198.28 | 0.06 | 1806.59 | 13.89 | 200.5 | 0.05 |
| ds2_a | 40 | 2.13E+08 | Infeasible | | 268.02 | 0.07 | 1804.17 | 31.24 | 269.7 | 0.07 |
| **ds2_a** | **50** | **2.13E+08** | **Infeasible** | | **325.17** | **0.08** | **1805.58** | **60.76** | **326.79** | **0.08** |
| **ds3_a** | **30** | **7.00E+08** | **Infeasible** | | **547.17** | **0.08** | **1811.09** | **61.62** | **552.16** | **0.08** |
| **ds3_a** | **40** | **7.00E+08** | **Infeasible** | | **723.15** | **0.17** | **1817.73** | **75.89** | **727.84** | **0.16** |
| **ds3_a** | **50** | **7.00E+08** | **Infeasible** | | **788.41** | **0.27** | **1812.8** | **82.43** | **791.78** | **0.27** |
| net_V | 10 | 333902.5 | 8.56 | 2.20 | 2.14 | 0.77 | 2.07 | 0.96 | 9.04 | 0.76 |
| net_V | 20 | 333902.5 | Infeasible | | 6.85 | 3.16 | 8.82 | 0.97 | 25.8 | 0.87 |
| net_V | 30 | 333902.5 | 4.35 | 10.24 | 5.23 | 5.60 | 1682 | 1.00 | 1690.56 | 0.99 |
| net_V | 40 | 333902.5 | 62.82 | 14.42 | 6.76 | 4.70 | 1801.53 | 4.90 | 1864.46 | 3.95 |
| net_X | 10 | 1141982.78 | Infeasible | | 16.82 | 1.20 | 65.93 | 0.84 | 29.5 | 0.90 |
| net_X | 20 | 1141982.78 | 29.97 | 3.99 | Infeasible | | 1801.73 | 1.63 | 1833.23 | 1.76 |
| **net_X** | **30** | **1141982.78** | **62.40** | **6.74** | **Infeasible** | | **1801.39** | **53.51** | **1864.24** | **3.74** |
| **net_X** | **40** | **1141982.78** | **82.88** | **10.39** | **Infeasible** | | **1801.88** | **77.02** | **1886.01** | **5.74** |

### 6.2 Conclusion and further scope

In this section we presented our results for the MILP model and the heuristics proposed. Using the heuristic to get a solution for ***FMIP*** model is useful as here we need to solve a sequence of LP problems. Rounding variables iteratively to solve the multi-modal supply chain MILP model exploiting its properties can be an effective approach to to get a good feasible solution within a time limit. This is an important aspect for practitioners as the model is intended for tactical and operational purposes.

The model presented here accounts for backlogs and lost sales, which makes the model more realistic in use. The ***FMIP*** model can also provide a base for the case where there are multiple commodities. This can be done by adding an additional index and constraints binding the commodities (e.g. constraints similar to that in 3.4 and 3.5).

Accommodating perishability and transfer times in the model would make the model more relevant for other applications where these aspects need to be considered. Additionally, devising an exact algorithm for the problem can also be pursued for further research.

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