LIFE PERIOD BASED COLLECTION AND INSPECTION MODEL IN CLOSED LOOP SUPPLY CHAINS

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Extended Abstract

Closed Loop Supply Chain (CLSC) has gained a widespread importance today, in the world of increasing environmental concerns and stringent regulations on the wastage caused right from inception of a product, through its life period and after it. A CLSC consists of the forward supply chain, (traditionally referred to as supply chain), and the reverse supply chain. The forward supply chain essentially involves the movement of goods/ products from the upstream suppliers to the downstream customers [1]. The reverse supply chain involves the movement of used / unsold products from the customer to the upstream supply chain, for possible recycling and reuses [2]. It has been found that reverse supply chains should be part of supply chain integrated, as it can contribute to lowering overall costs and meeting governmental/environmental regulations [3]. Hence there is an urgent to need to model and analyze closed loop supply chains as a system in total, without splitting into distinct parts of forward and reverse chains.

In this paper, the production and inventory planning model for the CLSC has been presented, considering the 'traditional' forward supply chain planning along with product recovery from the market, and the life period of the product. A three echelon CLSC is considered in this research comprising of Production Unit, Distributor, Consumer, Collection and Inspection, and Recycling Unit. Supplier and Alternate Market are assumed external to the CLSC (see Figure 1).

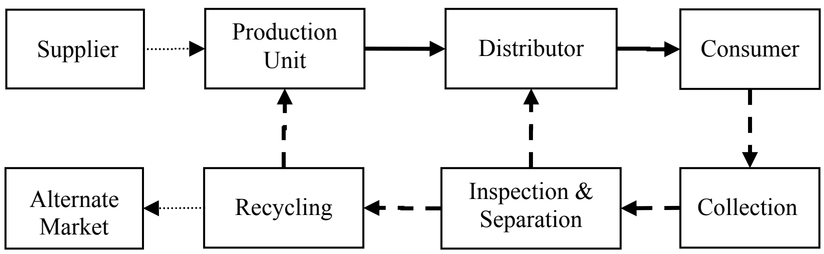

Figure 1: Closed Loop Supply chain (Adapted from [3])

The key contributions of this paper, is the explicit formulation of quantity and quality of returned products by the consumer, based on the product life cycle. The knowledge of what products to collect and what to reject right at the collection point with life period of a product as the criteria, helps save time and money. These models are then incorporated as part of the supply chains production and inventory planning model, to minimize the overall CLSC cost. In past literature, Savaskan *et al.* [4] has addressed the influence of incentives to the collection aspect of CLSC.

The Collection and Inspection (C&I) unit model formulation is based on the quality and quantity of the returned products. The objective function aims to minimize the costs associated with its inventory and backlogs associated with distributor and recycler demands. The cost incurred due to collection of products with life period beyond than that is profitable for the collector is also modeled.

The constraints in the C&I unit formulation consist of (1) inventory balance equations for products demanded by recycler and the distributor; (2) constraint to determines value of a binary variable used to capture the marginal costs associated with collection of products with life period more than that is profitable for the collector.

Modeling Quantity of Product Returns

The logic behind the formulation of the quantity of products collected from the consumers is based on capturing the willingness of the consumer to return the products in period t_2, which had been purchased in some past period t_1 ($t_1 < t_2$). Also, it is assumed that a consumer who bought a product in period $t_1 = t_2 - G$, where G is the lowest life period of used products that are profitable and feasible for collector to collect would be less willing to return, as compared to someone who bought it in period $t_1 = t_2 - J$, where $J > G$. In other words, consumers are assumed less likely to return products purchased in the recent past than in the distant past. In such case, Returns in period t_2, $R_{ci}(t_2)$ can be expressed as:

$$R_{ci}(t_2) = \sum_{t_1=t_2-J(t_2)}^{t_2-G} p(t_2,t_1)L(t_2,t_1) \quad \text{where } (t_1<t_2), \tag{1}$$

where, $p(t_2,t_1)$ is the probability of returning a product which was bought (t_2-t_1) periods ago; and $L(t_2\ ,t_1)$ is the number of used products sold in period t_1 which have not been returned by a consumer till period of collection t_2-1 and could be returned in period t_2. The upper and lower limits of the summations represent the periods when the products were sold, and these products are collected by the collector after being used in period t_2. $J(t_2)$ is the oldest life period of used products among the ones acquired by collector in period t_2. The value of $L(t_2,t_1)$ is captured by the following equations:

$$L(t_2,t_1) = L(t_2-1,t_1) - p(t_2-1,t_1)L(t_2-1,t_1) = [1-p(t_2-1,t_1)]L(t_2-1,t_1) \tag{2}$$

$$L(t_2,t_1) = D_c(t_1) \quad \textit{for initial } t_2 \ \ \& \textit{ for } t_2 - t_1 = G \textit{ thereafter} \tag{3}$$

As an example, consider the case when the probability distribution $p(t_2, t_1)$ is a linear function of the life period of the products (the difference between time when product is being returned, t_2 and time when it was bought, t_1) returned in period t_2 (see Figure 2). The probability of return is take values 0.5, 0.4, 0.3, 0.2 & 0.1 for $t_2 - t_1$ = 6, 5, 4, 3 and 2 respectively. (Note the decrease in probability as purchase period comes closer to return period). Also, it is noted that the discussion in this paper is valid even if the probability curve does not follow linear pattern. Depending on the benefits offered by the collector and also market conditions like upcoming new products, the curve may be convex or concave in nature. This formulation of quantity of returned products takes care of the fact that the summation of returns in future periods t_n to t_m of the products bought in certain past period t_1, is less than or equal to the demand in t_1:

$$D_c(t_1) \geq \sum_{t_2=t_n}^{t_m} p(t_2,t_1)L(t_2,t_1)$$

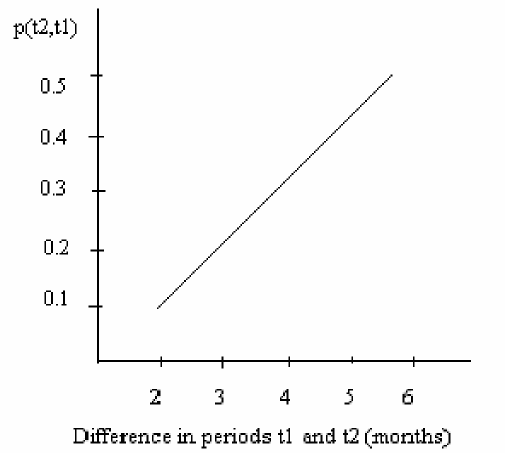

Figure 2: Probabilistic distribution of returns

The above formulation is further illustrated using the example consumer data shown in Table 1. Now, let the collections (for the first time) happen in period t_2 = 10, let G=2 and J(10)=6. Then from Equation 1,

$$R_{ci}(10) = \sum_{t_1=4}^{8} p(10,t_1)L(10,t_1)$$

Table 1: Consumer demand in periods 1 to 14

Period	1	2	3	4	5	6	7
Demand	10	11	17	13	13	15	16
Period	8	9	10	11	12	13	14
Demand	11	11	10	15	12	11	10

From Equation 3, $L(10,t_1)=D_c(t_1)$ as this is initial period when collection begins. Using Equations 1 and 3, the total products returned in period t_2 =10 = (6.5+5.2 +4.5+3.2+1.1) ≈ 20 products.

Now, for next period t_2=11, let G=2 & J(11)=5. Then from Equation 1, $R_{ci}(11) = \sum_{t_1=6}^{9} p(11,t_1)L(11,t_1)$

The probabilities $p(t_2,\ t_1)$ which depends on difference between the return period and the period in which the products were bought will remain same as shown in Figure 2. Using Equation 2, the value of $L(11,t_1)$ is calculated for t_1 = 6, 7, 8, 9 and 9, as shown:

L(11, 6) = [1 − p(10, 6)]L(10, 6) = 0.7×15 = 10.5
L(11, 7) = [1 − p(10, 7)]L(10, 7) = 0.8×16 = 12.8
L(11, 8) = [1 − p(10, 8)]L(10, 8) = 0.9×11 = 9.9
L(11, 9) = [1 − p(10, 9)]L(10, 9) = 11(since $t_2-t_1 = G = 2$)

Therefore, $R_{ci}(11)$ = (0.4×10.5 + 0.3×12.8 + 0.2×9.9 + 0.1×11) = 4.2+3.84+1.98+1.1≈ 11 products.

Modeling Quality of Product Returns

The quality of returns can be captured by the probability of product being reusable, $k(t_2, t_1)$ and probability of product being non-reusable but recyclable $q(t_2, t_1)$. Product reuse is defined as the use/ selling of returned products directly after cleaning operations, without any repair work, e.g. soft drink bottles. Recycling involves the reduction of returned products to the raw material components, leading to remanufacturing. These two probabilities, $q(t_2, t_1)$ and $k(t_2, t_1)$, are also defined as a function of the difference between time when product is being returned, t_2 and time when it was bought, t_1. However, as opposed to $p(t_2, t_1)$, the general trends of $q(t_2, t_1)$ and $k(t_2, t_1)$ is assumed to decrease as the difference increase. This is based on the assumption that lower the difference between t_2 and t_1, higher the probabilities of reuse and recycle. Also, the sum of $q(t_2, t_1)$, $k(t_2, t_1)$ and disposal probability equals 1.

These probability distributions, $q(t_2,t_1)$ and $k(t_2,t_1)$, are used to identify the value of $U(t_2)$, the oldest life period of used products profitable for the collector to acquire in period t_2. The value of $U(t_2)$ is required to determine the costs incurred by the C&I unit if products with life period more than $U(t_2)$ are collected, to satisfy demands from recycler and distributor. The value of $U(t_2)$, is calculated based on the concept of marginal benefits gained by the C&I unit in period t_2 for the collection of products with life period t_2–t_1. The marginal benefits of C&I unit is the difference between the revenues gained from the distributor and the recycler, and the costs incurred by C&I in terms of disposal, inspection costs and other incentives given to consumers. Now, the revenues are realistically assumed dependent on the quality and the quantity of the returned products, highlighting the use of distributions $q(t_2, t_1)$ and $k(t_2, t_1)$ in determining $U(t_2)$. The first value of t_1 for which the marginal benefit changes from being positive to negative (i.e. costs > revenue) is determined, which is used to fix $U(t_2)= t_2 - t_2 -1$.

The factors affecting the value of $J(t_2)$ are the values of demand from the distributor and from recycler. Let $U_1(t_2)$ and $U_2(t_2)$ denote the oldest life periods of products, collection of which will satisfy demand from distributor and recycler respectively. Then, depending on relation between $U(t_2)$, $U_1(t_2)$ and $U_2(t_2)$, the value of $J(t_2)$ is determined so as to minimize the overall costs in the objective function. As the value of $J(t_2)$ determines the actual number of products collected in period t_2 (see Equation 1) it is also used in the formulation of inventory balance equations for products demanded by recycler and those demanded by the distributor. The value of $J(t_2)$ is also used to determine value of a binary variable which affects the marginal costs part in the objective function. Also, it is used in determining the quantities shipped to both recycler and distributor in constraints. The complete C&I optimization model is shown below.

$$Minimize\ z = \sum_{t_2=1}^{T} \left\{ \begin{array}{l} C_{bp}\left[S_{wi}(t_2)+S_{ri}(t_2)\right]+C_{hp}I_{ri}(t_2)+C_{hp}I_{wi}(t_2)+ \\ \mu \cdot F(t_2,t_1)\left[\displaystyle\sum_{t_1=t_2-J(t_2)}^{t_2-G} p(t_2,t_1)L(t_2,t_1)-\sum_{t_1=t_2-U(t_2)}^{t_2-G} p(t_2,t_1)L(t_2,t_1)\right] \end{array} \right\}$$

$$where\ F(t_2,t_1)=[(C_d+C_i+b(t_2,t_1))-[k(t_2,t_1)(C_d+V_{wi})+q(t_2,t_1)(C_d+V_{ri})]]$$

Subject to

$$S_{wi}(t_2)-I_{wi}(t_2)=\underbrace{D_{wi}(t_2)+S_{wi}(t_2-1)}_{\text{Net\_demand(Reuse)}}-\underbrace{\left\{I_{wi}(t_2-1)+\sum_{t_1=t_2-J(t_2)}^{t_2-G} k(t_2,t_1)p(t_2,t_1)L(t_2,t_1)\right\}}_{\text{Inventory\_on\_hand(Reuse)}} \quad \text{(C1)}$$

$$S_{ri}(t_2)-I_{ri}(t_2)=\underbrace{D_{ri}(t_2)+S_{ri}(t_2-1)}_{\text{Net\_demand(Recycle)}}-\underbrace{\left\{I_{ri}(t_2-1)+\sum_{t_1=t_2-J(t_2)}^{t_2-G} q(t_2,t_1)p(t_2,t_1)L(t_2,t_1)\right\}}_{\text{Inventory\_on\_hand(Recycle)}} \quad \text{(C2)}$$

$$\left[J(t_2)-U(t_2)\right]-M\cdot\mu\le 0\ \ (For\ some\ large\ M) \quad \text{(C3)}$$

The objective function for C&I Unit aims to minimize the costs associated with its inventory and backlogs, and also captures the cost of collection of products with life period more than that is profitable for the collector, where C_{hp} is the inventory holding cost per unit, C_{bp} is the backlog cost per unit, C_d is the disposal cost per unit, C_i is the inspection cost per unit, V_{ri} is the per unit revenue from Recycler, V_{wi} is the per unit revenue from Distributor and $b(t_2, t_1)$ is the per unit incentive given to the consumers. Constraint (C1) represents the inventory balance equation for the reusable products demanded by the Distributor where I_{wi} is inventory of reusable products, the S_{wi} is the backlog of reusable products, and D_{wi} is the demand for reusable products from Distributor. Constraint (C2) represents the inventory balance equation for the recyclable products demanded by the Recycler where I_{ri} is inventory of recyclable products, the S_{ri} is the backlog of recyclable products, D_{ri} is the demand for recyclable products from Recycler. Constraint (C3) captures the marginal costs incurred when $J(t_2)$ is greater than $U(t_2)$ as explained earlier, using the binary variable μ.

CLSC Production Planning Model

The overall CLSC objective function minimizes the sum of Production Unit costs, Distributor costs, C&I Unit costs and Recycler costs over a span of T periods. The objective function part concerning the Production Unit minimizes the sum of production costs, inventory holding costs (product and raw materials), backlog costs and ordering costs. The objective function part concerning the Distributor minimizes the costs related with holding and backlog of products and the ordering costs. The objective function for Recycler aims to minimize the costs associated with the inventory and the backlog associated with alternate market and demands from the production unit.

Now, apart from the C&I Unit constraints, the objective of CLSC is subject to a number of constraints laid down by other members of CLSC. The Production Unit constraints consist of the inventory balance equations both for finished products as well as raw materials, a constraint determining the demand to recycler assuming periodic review policy, constraint to determine amount shipped to the distributor and finally a constraint capturing bottleneck production rate. The Distributor constraints consists of an inventory balance equation and a constraint to relate demand from distributor to Production Unit and that to inspection unit while assuming a periodic review policy. Finally, the Recycler constraints consist of an inventory balance equation, taking into account common inventory storage for all the markets and an equation determining demand to C&I Unit assuming a periodic review policy. The full model is not presented due to space restrictions.

Conclusion

A centralized CLSC inventory planning model has been formulated. The critical aspects of the collection of used products from the market based on the product lifecycle, and quality aspects have been explicitly dealt with. Future work will involve the testing of the proposed model with realistic data, and the sensitivity analysis of the model with respect to the probability distributions of the returns.

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