Effects of Information Synchronization Frequency on the Stability of Supply Chains

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Abstract

With modern supply chains hurtling towards 'information overloading,' it is of great interest to know *what* data is required for effective decision making and *how often* the data needs to be updated. The latter part on *how often* is the focus of this paper. The inventory and production ordering policies among the players (Manufacturer and Distributors) of a two echelon supply chain are studied. System dynamics models of the different players are developed. *Z*-transform techniques are employed to derive the general stability condition. Stability of the supply chain under different strategies (communicative vs. vendor managed inventory) is appraised and their implications on the information synchronization needs are analyzed.
Keywords: Supply Chain Dynamics, Sampling Interval, z-Transformation, Stability Analysis

1. Introduction

Analysis of the dynamics of many supply chains (especially those with improper policies) reveals periods of inventory build-ups, stock-outs, overtime production and production shutdown, which costs the company in terms of profits. A natural choice to examine the supply chain dynamics is the application of control theoretic techniques. This often involves capturing of the supply chain system using feedback-based structures and analysis of the system through the application of control theoretic tools such as block diagram algebra, Bode plots, and functional transformations. A comprehensive review of literature on the use of control theoretic technique for analysis of supply chain can be found in [1]. In this research, a two-echelon supply chain system consisting of a Manufacturer and Distributor is modeled as system dynamic models and analyzed using control theoretic techniques. The models developed in this research improve over the past works of [2], [3] and [4] by the explicit representation of the frequency of information update, through the use of sampling interval, to study its effect on the system stability. Two types of information update are identified: intra-player and inter-player. The variation in the supply chain's stability in response to the information update frequency is examined. Also, the stability of the supply chain under different strategies (communicative vs. vendor managed inventory) is appraised and their implications on the information synchronization needs are expounded. Previously, [5] examined and confirmed the effect of improper sampling interval selection on the stability of collaborative supply chains.

2. Communicative Supply Chain Model

In communicative supply chain, each player makes independent decisions; hence the decision making at each individual player/ echelon is separately modeled using system dynamics. The end customer places orders to and receives goods from the Distributor, who in turn places orders to and receives goods from the Manufacturer, who produces the goods. The Distributor forecasts its sales, and based on its current inventory and goods-in-transit levels, places an order with the Manufacturer which is fulfilled after a fixed transportation delay. The Manufacturer, upon receiving the order from the Distributor, immediately dispatches the required quantities. Also, the Manufacturer uses its current work-in-process and end product inventory, and determines the production release quantities to its shop floor. The underlying equations for the Manufacturer and Distributor are described as follows.

The Manufacturer is assumed to forecast demand (*FD*) of its products based on a first order exponential smoothing of the dispatches (*DIS*), with an exponential smoothing constant of ρ.

$$FD_t = FD_{t-1} + \rho \cdot \delta \cdot (DIS_t - FD_{t-1}) \tag{1}$$

Here, δ (1 day – 1 week) is the sampling interval for the Manufacturer's model. The sampling interval is said to correspond with the frequency at which information is updated internally and externally to the player. Typically, in past research work ([4], [6]), the sampling intervals are implicitly assumed to be equal between members and are set at 1, indicating a weekly update of the ordering rule.

The Manufacturer's inventory (*MINV*) accumulates the difference in the product production (*PROD*) and product dispatch rates (*DIS*), while the work-in-process (*WIP*) accumulates the differences in the production release (*PREL*) and production (*PROD*).

$$MINV_t = MINV_{t-1} + \delta \cdot \left(PROD_t - DIS_t \right) \tag{2}$$

$$WIP_t = WIP_{t-1} + \delta \cdot \left(PREL_t - PROD_t \right) \tag{3}$$

The production release quantities (*PREL*) are determined by based upon the forecasted demand (FD), difference between the desired level of WIP and the current WIP, and difference between the desired level of inventory and the current inventory (*MINV*). Based on Little's Law, the desired WIP levels are set to yield the desired throughput (forecasted demand), given the manufacturing lead time (*L*); and the desired inventory is set equal to the forecasted demand. Also, the production rate (*PROD*) is modeled as a pipeline delay of the production releases with a lead time of *L*.

$$PREL_t = FD_{t-1} + \alpha \cdot (L \cdot FD_{t-1} - WIP_{t-1}) + \beta \cdot (FD_{t-1} - MINV_{t-1}) \tag{4}$$

$$PROD_t = PREL_{t-L} \tag{5}$$

The fractional adjustment rate for WIP (α) and inventory (β) describes the rate at which the shortfall between the desired and current levels of WIP and inventory and corrected, respectively. For instance, $\alpha=0$ completely ignores the work-in-process, while $\alpha=1$ fully accounts for the work-in-process. Selection of these adjustment rates reflects the ordering nature of the firms, and hence are the decision variables.

The Distributor's model is assumed to be identical to the Manufacturer's model (Equations 1-5), with τ as its exponential smoothing constant, ψ as the fractional adjustment rate for goods-in-transit (*GIT*), φ as the fractional adjustment rates of the distributor inventory (*DINV*), *R* as the transportation lead time and Δ as the sampling interval (1 day – 1 week) for the Distributor's model. Also, *FDD* denotes the forecasted demand, *DSALES* denotes the end customer sales rate, *DEL* denotes the product delivery rates of the distributor

$$FDD_t = FDD_{t-1} + \tau \cdot \Delta \cdot (DSALES_t - FDD_{t-1}) \tag{6}$$

$$DINV_t = DINV_{t-1} + \Delta \cdot \left(DEL_t - DSALES_t \right) \tag{7}$$

$$GIT_t = GIT_{t-1} + \Delta \cdot \left(DIS_t - DEL_t \right) \tag{8}$$

$$DIS_t = FDD_{t-1} + \psi \cdot (R \cdot FDD_{t-1} - GIT_{t-1}) + \varphi \cdot (FDD_{t-1} - DINV_{t-1}) \tag{9}$$

$$DEL_t = DIS_{t-R} \tag{10}$$

3. System Analysis using z-Transform Techniques

3.1 Z-transform Description

In this paper, the *z*-transform technique is used to obtain the generalized transfer function and later for the stability conditions for the Manufacturer and Distributor. The z-transforms of the Manufacturer's production ordering and inventory management system described in Equations (1)-(5) are given as follows:

$$FD[z] = \frac{\delta \cdot \rho \cdot DIS[z]}{z - 1 - \delta \cdot \rho} \tag{11}$$

$$MINV[z] = \frac{z \cdot \delta \cdot (PROD[z] - DIS[z])}{z - 1} \tag{12}$$

$$WIP[z] = \frac{z \cdot \delta \cdot (PREL[z] - PROD[z])}{z - 1} \tag{13}$$

$$PREL[z] = \frac{(1 + \alpha \cdot L + \beta) \cdot FD[z] - \alpha \cdot WIP[z] - \beta \cdot MINV[z])}{z} \tag{14}$$

$$PROD[z] = z^{-L} \cdot PREL[z] \tag{15}$$

Z-transform of the exponential smoothing forecast function (Equation 1) is as shown in Equation (11). Equations (12) and (13) represent the z-transforms of the Manufacturer's inventory and WIP respectively, obtained using the Heaviside Step Function or the integration term $1/(1-z^{-1})$. The z-transforms of the production release ordering and production rate are given in Equations (14) and (15), respectively. Using algebra, the general transfer function for *PREL/DIS* has been obtained by solving Equations (11)-(15) simultaneously, to yield Equation (16).

$$\frac{PREL[z]}{DIS[z]} = \frac{z^L \delta\left(\left(z-1\right)\left(1+L\alpha\right)\rho + \beta\left(z-1-\rho+z\rho+\delta\rho\right)\right)}{\left(z^{L+1}+\left(\beta-\alpha\right)\delta+z^L\left(\alpha\delta-1\right)\right)\left(z-1+\delta\rho\right)} \tag{16}$$

For the remainder of this analysis, the exponential smoothing parameter ρ is fixed at an arbitrary value of 0.2; and in order not to solve a transcendental function, the value of production lead time L is set at 3 weeks. Substituting in Equation (16), the transfer function *PREL/DIS* is reduced to:

$$\frac{PREL[z]}{DIS[z]} = \frac{z^3\delta\left(z-1-3\alpha+3\alpha z-6\beta+6\beta z+\beta\delta\right)}{\left(z^4+\left(\beta-\alpha\right)\delta+z^3\left(\alpha\delta-1\right)\right)\left(5z-5+\delta\right)} \tag{17}$$

Distributor's general transfer function for *DIS/DSALES* can be obtained in the similar fashion as outlined above to yield a function identical to Equation (16) albeit in terms of τ, ψ, φ, Δ and R. For the remainder of this analysis, it is assumed that the Distributor's smoothing constant $\tau = 0.2$ and transport lead time $R = 2$ weeks, which yields DIS/DSALES as:

$$\frac{DIS[z]}{DSALES[z]} = \frac{z^2\Delta\left(z-1-2\psi+2\psi z-6\varphi+6\varphi z+\varphi\Delta\right)}{\left(z^3+\left(\varphi-\psi\right)\Delta+z^3\left(\psi\Delta-1\right)\right)\left(5z-5+\Delta\right)} \tag{18}$$

3.2 Stability Analysis and System Responses

It is essential to understand how the supply chain responds to any change in its input (i.e. demand), especially under a fluctuating market. Does the response result in increasing amplitude oscillations and chaos in general, or does the response appear controllable and damped? Under what conditions is the system stable or unstable? A system given by its closed form transfer function is said to be stable if all the roots (poles) of the function's denominator polynomial is within the unit circle in the complex plane [7]; unstable if the poles are outside the unit circle or repeated poles on the circle; critically/ marginally stable if non-repeating poles are there on the unit circle. The roots of the numerator polynomial (zeros) represent the roots of the feed forward part of the transfer function of a system. There is no restriction on the values of zeros other than that required to obtain a desired frequency or impulse response.

As denominator polynomial of the transfer functions is of a higher order, it is desirable to test the location of the roots on the complex z-plane without explicitly solving for the roots. In this research, Jury's Test [7] is employed to determine the location of the roots. Though this method enables a solution, it still involves tedious calculations, which are hence performed by the authors by using Mathematica®.

Now, the conditions for stability of the Manufacturer for particular setting of sampling intervals are derived, and sample simulation results are presented. Substituting the sampling interval δ equal to 1 week in Equation (16), expanding the denominator polynomial and employing Jury's Test reveals the stability conditions in terms of α and β. The stability conditions are illustrated on the parameter plane, as shown in Figure 1A where each curve illustrates a stability condition. The system is guaranteed to be stable when the values of α and β are restricted to the stable region. Similarly, the conditions for stability of the Distributor in terms of ψ and φ are derived and are as illustrated in Figure 1B.

Though the stability regions of Manufacturer and Distributor are distinct, the stability of the supply chain as a whole is dependent upon the interaction between the Manufacturer and the Distributor. Also, given the fact that the supply chain is a serial system with the output of the Distributor (DIS) feeding in as the input to the Manufacturer, any instability of the Distributor will propagate backwards onto the Manufacturer. Intuitively, when the Manufacturer observes oscillatory growth in its production release, it can be either due to the faulty setting of its control parameters or the propagation of disturbances from downstream. However, it is noted that the instability in Manufacturer does not propagate downstream for the system under study. Although, the above observation can be easily seen using simulation experiments, such experiments are not presented in this paper.

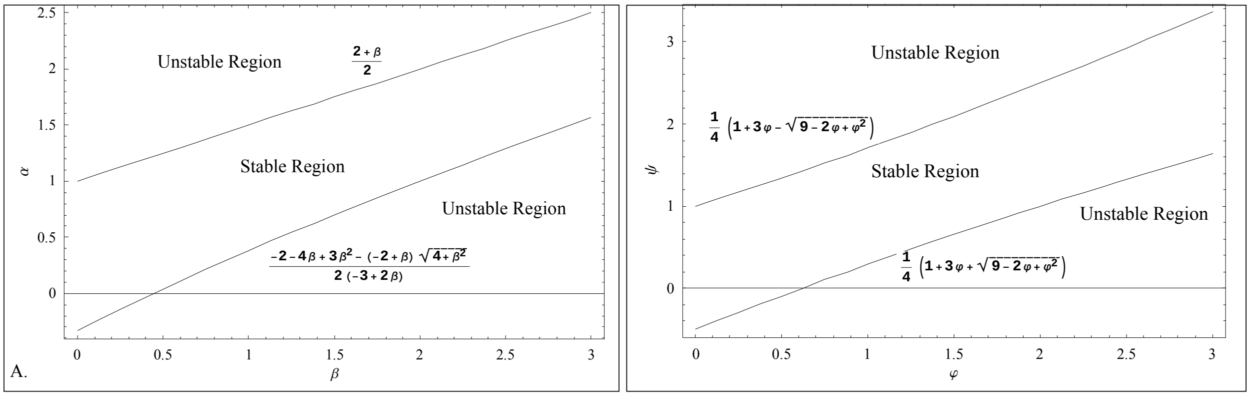

Figure 1: Stability regions of (A) Manufacturer in the α - β plane; (B) Distributor in the ψ - φ plane

4. Analysis of Vendor Managed Inventory (VMI) Supply Chain

In a VMI configuration, the Manufacturer and Distributors cooperate with each other, sharing resources and together plan and execute operations. Under the VMI scheme considered in this research, the Distributor sends their current inventory level and end customer sales data to the Manufacturer. The Manufacturer determines the quantity of goods to be dispatched to the Distributor. The distribution ordering policy as such is assumed not to change. That is, the Manufacturer assumes the same policy as employed by the Distributor in determining the dispatch order quantities. Now, the Manufacturer will perform forecasting of Distributor's demand, maintenance of GIT, and calculation of dispatch quantity. This is reflected by modifying Equations (6), (8), (9) and (10), replacing Δ by δ, since δ is the sampling interval of the Manufacturer (see Equations 19-22). The maintenance of inventory (DINV) is still performed by the Distributor and hence the use of sampling interval Δ in Equation (23).

$$FDD_t = FDD_{t-1} + \tau \cdot \delta \cdot (DSALES_t - FDD_{t-1}) \tag{19}$$

$$GIT_t = GIT_{t-1} + \delta \cdot \left(DIS_t - DEL_t \right) \tag{20}$$

$$DIS_t = FDD_{t-1} + \psi \cdot (R \cdot FDD_{t-1} - GIT_{t-1}) + \varphi \cdot (FDD_{t-1} - DINV_{t-1}) \tag{21}$$

$$DEL_t = DIS_{t-R} \tag{22}$$

$$DINV_t = DINV_{t-1} + \Delta \cdot \left(DEL_t - DSALES_t \right) \tag{23}$$

The production order (Equations 1-5) and VMI-based dispatch ordering (Equations 19-23) are still independent sub-systems. Hence the stability region of *PREL* in terms of α and β will remain unaffected (same as shown in Figure). Now, the impact of the frequency of information update on the dynamics of the supply chain systems needs to be explicitly measured. Typically in the past research works, the frequency at which the Distributors send their demand and inventory information to the Manufacturer is the same as the frequency at which the Manufacturer makes decisions (e.g. every week). In this section, the effect of different settings of the sampling intervals δ and Δ on the stability conditions in terms of ψ and φ are analyzed. That is, what happens when the Manufacturer updates its information every day but the Distributors send their updated information every week?

Method similar to the one described in Section 3.1, z-transforms of Equations (19)-(23) are carried out and the general transfer function for *DIS/DSALES* are obtained (τ = 0.2, *R = 2* weeks):

$$\frac{DIS[z]}{DSALES[z]} = \frac{z^2\left(5(z-1)\varphi\Delta + \delta\left(-1-2\psi-\varphi+z(1+2\psi+\varphi)+\varphi\Delta\right)\right)}{\left(z^3-\psi\delta+z^2(-1+\psi\delta)+\varphi\Delta\right)(5z-5+\Delta)} \tag{24}$$

The presence of δ and Δ in the denominator polynomial indicates that the stability of dispatch depends on both the sampling interval of the Manufacturer and Distributor. That is, both the frequency at which information is obtained from the Distributor and the frequency at which the decision is made by the Manufacturer affect the dispatch ordering stability. The stability conditions *DIS/DSALES* are obtained in terms of ψ and φ for each of the following cases: δ=1 and Δ=1, δ=1 and Δ=1/7, δ=1/7 and Δ=1, δ=1/7 and Δ=1/7 using Jury's Test. An interval of 1 corresponds to update of information every week, and 1/7 corresponds to update of information every day.

The stability conditions for the different settings of the sampling interval are plotted in the $\psi - \varphi$ plane, as shown in Figures 2A and 2B. In Figure 2, curves on the same pattern (solid, or dashed and solid thick lines) represent the stability conditions for a particular setting of the sampling interval. For each setting, the stable region is between the corresponding curves (and above the axis $\psi = 0$).

It is observed from Figure 2A that the region of stability under VMI when sampling interval equals to 1 week, is the same as when the dispatch ordering was done by the Distributor under communicative supply chain, as seen in Figure 1B. Higher values of ψ and φ allows for an aggressive ordering policy. Now, the sampling interval $\delta=1$, $\Delta=1/7$ indicates that the Distributor provides updated inventory and demand data on a daily basis, while the decisions at the Manufacturer are taken weekly. The stability region for this case is shown in Figure 2A, enclosed by the dashed lines. It is seen that the stability region has flattened, allowing for higher values of φ (fractional adjustment rate of Distributor inventory) while limiting ψ (fractional adjustment rate of goods in transit). The Manufacturer can now aggressively account for shortfalls in Distributor inventory.

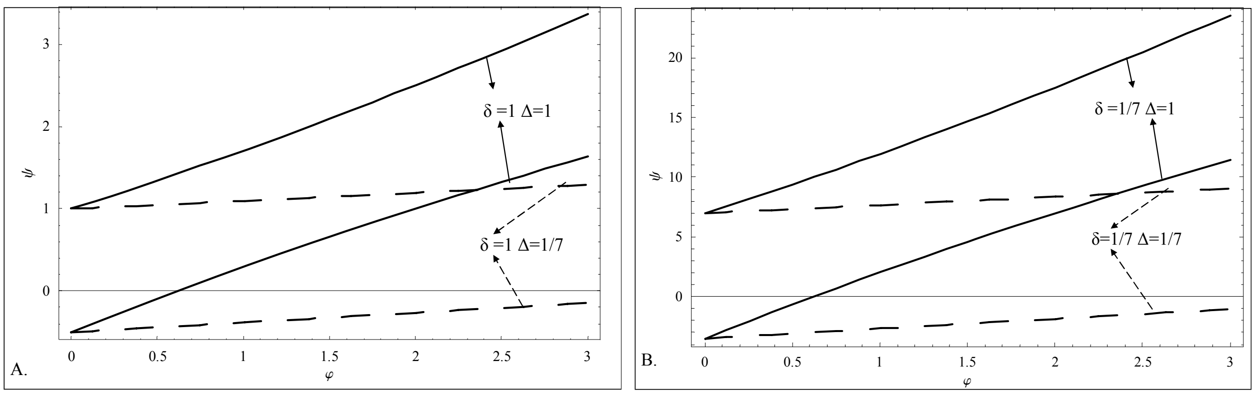

Figure 2: Stability regions under VMI policy in the ψ - φ plane: (A) $\delta >= \Delta$ (B) $\delta <= \Delta$

The stability regions for the case of $\delta=1/7$, $\Delta=1$ and $\delta=1/7$, $\Delta=1/7$ are shown in Figure 2B. Though the curves in Figure 2B look similar to those in Figure 2A, it is noted that the Y-axis range in Figure 2B is from 0 to 20 while the Y-axis range in Figure 2A is from 0 to 4. For the sampling interval $\delta=1/7$, $\Delta=1$, the stability region is enclosed by the solid line. It is seen that increase in the frequency of updates at the Manufacturer ($\delta=1/7$), allows for higher range of values for ψ, aggressively accounting for the GIT. Now, for the sampling interval $\delta=1/7$, $\Delta=1/7$, the stability region is enclosed by the dashed line. It is seen that this provides the biggest area of stability region since both data and decisions are updated are more frequent basis (i.e. daily).

Hence, the aggressiveness in the dispatch ordering has to be balanced by the corresponding investment in getting frequent updates of information. Conversely, with increasingly frequent availability of information, the supply chain can move towards a more aggressive ordering policy. It is noted that the information obtained from the Distributor and the internal Manufacturer information are assumed to be the accurate information. The effects of updates frequency of inaccurate information is left as future research.

5. Improved Production Ordering with VMI

To take advantage of the improved availability of information through the VMI scheme a new production ordering mechanism is analyzed, where the Manufacturer's production release order (*PREL*) is calculated based on the total supply chain inventory and the more accurate forecast obtained from the Distributor's sales data. Now, the VMI part of dispatch ordering remains the same as in Equations (19)-(23). The underlying equations of the Manufacturer are as shown in Equations (25)-(29). Equations (25), (26) and (27) remain the same as in Equations (2), (3) and (5). A new variable total WIP (*TWIP*) is introduced, which is the sum of the Manufacturer's WIP, inventory (*MINV*) and goods-in-transit (*GIT*). This TWIP is used in the calculation of *PREL* (Equation 29). Also, in Equation (29), ω represents the fractional adjustment rates of total WIP, while φ represents the fractional adjustment rates of the Distributor's inventory.

$$MINV_t = MINV_{t-1} + \delta \cdot \left(PROD_t - DIS_t \right) \tag{25}$$

$$WIP_t = WIP_{t-1} + \delta \cdot \left(PREL_t - PROD_t \right) \quad (26)$$

$$PROD_t = PREL_{t-L} \quad (27)$$

$$TWIP_t = WIP_t + MINV_t + GIT_t \quad (28)$$

$$PREL_t = FDD_{t-1} + \omega \cdot ((L+R) \cdot FDD_{t-1} - TWIP_{t-1}) + \varphi \cdot (FDD_{t-1} - DINV_{t-1}) \quad (29)$$

Considering the supply chain as a whole, the *z*-transforms of Equations (19)-(23), and Equations (25)-(29) are obtained. Similar to steps outlined in Section 3.1, the general transfer function *PREL/DSALES* is calculated. Since for performing stability analysis the denominator polynomial of *PREL/DSALES* is of importance, it is shown in Equation (30) (L=3, R=2, τ=0.2).

$$\left(\left(z - 1 + \omega\delta \right) \left(5z - 5 + \Delta \right) \left(z^3 + \Delta\varphi - \delta\psi + z^2 (-1 + \delta\psi) \right) \right) \quad (30)$$

It is observed from Equation (30) that the stability of the supply chain system as a whole is dependent upon ω, ψ, φ, δ and Δ. Now using Jury's Test, the stability conditions *PREL/DSALES* and *DIS/DSALES* are obtained. Stability of *DIS/DSALES* will be exactly the same as discussed in Section 4. Stability conditions *PREL/DSALES* obtained in terms of ω and ψ for each of the following cases: (φ=1, δ=1, Δ=1), (φ=1, δ=1 Δ=1/7), (φ=1, δ=1/7, Δ=1), (φ=1, δ=1/7, Δ=1/7); and in terms of ω and φ for each of the following cases: (ψ=1, δ=1, Δ=1), (ψ=1, δ=1 Δ=1/7), (ψ=1, δ=1/7, Δ=1), (ψ=1, δ=1/7, Δ=1/7). It was observed that for all cases with δ=1 the roots turns out to be ω=0 and ω=2; and for δ=1/7 the roots turns out to be ω=0 and ω=14. That is, ψ and φ are bounded by the stability region as obtained in Section 4, and ω can be varied independent of ψ and φ within the range specified. This implies that once the dispatch ordering (*DIS*) has been tuned based on the supply chain settings, the performance of the supply chain can be solely controlled by the fractional adjustment rate of total WIP, ω.

6. Conclusions

The information synchronization effects in a two echelon supply chain operating under (1) communicative settings, (2) VMI settings and (3) VMI with improved ordering has been analyzed. Generalized stability conditions have been derived using *z*-transform technique. The differences in the frequency of information update frequency at the different players and their effect on the overall system stability has been analyzed. In case of communicative supply chain structure, the Distributor and Manufacturer forms independent sub-systems, and hence the have distinct stability regions, with no influence of their individual information update frequency on the other player. In the case of VMI supply chain, the aggressiveness in the dispatch ordering has to be balanced by the corresponding investment in getting frequent updates of information. Conversely, with increasingly frequent availability of information, the supply chain can move towards a more aggressive ordering policy. Under VMI with improved ordering, the fractional adjustment rate of total system inventory can be varied independent of the VMI's dispatch ordering. This implies that once the dispatch ordering (*DIS*) has been tuned based on the supply chain settings, the performance of the supply chain can be solely controlled by the fractional adjustment rate of total WIP.

References

[1] Ortega, M., and Lin, L., 2004, "Control Theory Applications to the Production-Inventory Problem: A Review," International Journal of Production Research, 42(11), 2303-2322.

[2] Forrester, J.W., 1961, Industrial Dynamics, MIT Press, Cambridge.

[3] John, S., Naim, M.M., and Towill, D.R., 1994, "Dynamic analysis of a WIP compensated decision support system," International Journal of Manufacturing System Design, 1(4), 283-297.

[4] Disney, S.M., and Towill, D.R., 2002, "A Discrete Transfer Function Model to Determine the Dynamic Stability of a Vendor Managed Inventory Supply Chain," International Journal of Production Research, 40(1), 179-204.

[5] Venkateswaran, J., and Son, Y., 2005, "Information Synchronization Effect on the Stability of Collaborative Supply Chain," Proc. of the Winter Simulation Conference, Orlando, Florida.

[6] Grubbström, R.W., 1998, "A net present value approach to safety stocks in planned production," International Journal of Production Economics, 56(7), 213–229.

[7] Jury, E. I., 1964, Theory and Application of the z-Transform Method, Robert E. Krieger, New York.