Queuing in Space: Design of Message Ferry Routes in Static Adhoc Networks

Veeraruna Kavitha and Eitan Altman
Kavitha.Voleti_Veeraruna@sophia.inria.fr, Eitan.Altman@sophia.inria.fr

***Abstract*—We study the concept of Ferry based Wireless Local Area Network (FWLAN), in which a number of isolated nodes are scattered over some area and where communication between a node and the outer world, or communication between the nodes, are made possible via a message ferry. The Ferry has a predetermined cyclic path which collects messages from a node and delivers messages to it when it is in the vicinity of the node. We use the mathematical theory of polling systems to study the performance of the FWLAN. We consider three different architectures and each one of them is mapped to an appropriate polling system. The polling disciplines that are needed for modeling the FWLAN involve non-standard variants of gating disciplines. Our goal is to design the routes of the Ferry as well as the points where it should stop to distribute and collect messages. This mathematical modeling brings another dimension to the classical related vehicle routing problem due to the radio channel: the cyclic path of the ferry need not touch every node. The distance between the node and the fairy at the point when communication occurs determines the transmission rate and hence the service time and thus the system's capacity.**

I. Introduction

Message Ferry are mobile relays or mobile base stations (BS) that serve as "postman" to deliver to static or dynamic wireless nodes messages (or packets) and to collect messages from them. Mobile BS have been proposed in the context of mobile Ad Hoc Networks [18], of Vehicular Ad-Hoc Networks (Vanets) [14] and of wireless (static) sensor networks [15]. In the UmassDiesel project, computers have been installed in 30 out of 40 buses and these then serve as Message Ferry to deliver messages to throw boxes (see http://prisms.cs.umass.edu/diesel/).

In this paper we are concerned with a message ferry that serves as a mobile access point in a local area network which we call FWLAN (Ferry Wireless LAN); the ferry delivers and collects messages from static nodes on some geographic area Δ. This problem is close in nature to the well studied vehicle routing problem, where a central point receives service requests from points on the plane. One or several service vehicles are then sent for handling the requests and one has to design efficient optimal vehicle's route that pass through all the points, see [4], [13], [11].

Beyond the resemblance of the vehicle routing problem to the problem of designing a route for a message ferry, we notice also a fundamental difference. In the vehicle routing problem, the routes need to pass **through** all the points in space that require service. In contrast, the routes of the message ferry need only pass in the vicinity of nodes (in their transmission and reception range). Moreover, the range itself is flexible: assuming a fixed transmission power, the range can be increased at the cost of decreasing throughput. The relation between the range and the throughput are determined by the radio propagation conditions.

Taking into account the radio conditions, we are concerned with the design of a cyclic route of the ferry and of the location of stops along the route. Each stop is assigned an area that contains all points closer to that stop than to other stops. When reaching a stop, the ferry collects (uplink) and dumps (downlink) data from/to the area assigned to it.

The larger the number of stops, the more time the ferry has to spend for stopping and accelerating. The lesser the number of stops is, the larger are the distances at which it has to receive/transmit the data from/to the nodes and hence the larger the time to achieve reliable communication is. In addition to this trade-off which appears both in the one dimensional and the two dimensional cases, there is another important design issue that is specific to the plane: in order to achieve smaller service times the routes need to be longer, which may increase waiting times. Larger service times that would be needed if the routes were shorter imply smaller achievable throughput of the system.

We consider three architectures:

- **Sensor Access Network (SAN):** there is a fixed base station (BS) that is connected to the global Internet (or to other BS) and thus enables communication between nodes in the FWLAN and the outer world. The ferry brings all traffic from (respectively to) nodes in the FWLAN to (resp. from) the BS. There is no traffic from nodes of the FWLAN to other nodes in the FWLAN.
- **Hybrid Access Network (HAN):** Same as SAN but traffic sent by a node in the FWLAN can also be destined to another node in the FWLAN. In that case the ferry first brings all uplink messages to the BS and then receives from the BS all downlink messages received during the last cycle including those just brought by the shuttle destined to other nodes in Δ. Using this architecture we can achieve routing within the area also, however it always takes two cycles to complete the data transfer.
- **Autonomous Network (AUN):** the Ferry serves as a local mobile BS. Thus a message sent by a node is transferred directly to the destination node without first transmitting through a fixed BS. In this case, in contrast to the HAN architecture, the data routing can take place faster: depending on the location of the source destination nodes and the direction of the ferry, a message may arrive

at the destination in the same cycle of the Ferry.

This kind of system can best be studied using a polling system, wherein a ferry serves a finite number of queues in a cyclical order [3], [7], [5], [10], [17], [16]. Two queues are considered at each stop. The uplink arrivals from nodes that are nearest to the stop under consideration are modeled as one queue (uplink queue) while all the nearest downlink arrivals are modeled as the downlink queue.

Some of the polling disciplines that are needed for modeling the downlink of the FWLAN involve non-standard variants of gating disciplines. In the SAN architecture, we note that upon arriving at a queue, the ferry serves (brings to the queue) all messages that were present at the BS when the ferry last arrived at the BS. If only downlink traffic existed then this would correspond to the "globally gated" (GG) discipline [7]. In contrast, when the ferry arrives at a stop it can upload all the traffic present there upon arrival (so that the standard gated or exhaustive disciplines can be used to model this). In the AUN architecture, the polling discipline is a combination of the synchronized gated regime introduced in [10] together with rerouting introduced in [16].

In designing the ferry's routes and stops, we aim at minimizing the expected waiting times at all nodes in the Pareto sense[1].

The system, model and the notations of the paper are introduced in Section II. We consider SAN and HAN architectures respectively in Sections III and VI. Section V treats a general network of which AUN is a special case. The theoretical results obtained were simulated using some numerical examples in the respective sections itself. The paper is concluded in Section VII.

II. System Model and Notations

We consider a 1-Dimensional or 2-Dimensional geographical area Δ in which static nodes (sensors) are scattered. We assume that the network is sparse and there is no direct global connectivity. In order to receive messages from the nodes or to send messages to them, a ferry called "message ferry" or "message shuttle" moves around and serves as a postman. The nodes either generate data to or acquire data form nodes within and/or outside the area. In order to route the data to and from outside the area, the shuttle has to pass through a BS that serves as a gateway.

It is possible that the BS is also needed to route the data within the area (for example in HAN architecture). In Section III on SAN architecture, all the data routing takes place via the BS: the ferry goes to the BS once in every cycle to collect and deposit the information. The ferry "serves" the nodes at each stop in a cyclic manner. We use throughout terms from queuing theory; by "serves" a message we mean that the ferry transmits it if the connection is downlink (i.e. the message is destined to a node), or receives it, if it is an uplink message. In Section V on AUN architecture, we consider instead the situation where the server/ferry routes the data directly to the destination, if an uploaded message was meant for a node within the FWLAN. Only the messages from/to the nodes outside Δ are routed to BS. Finally in Section VI, we give a brief idea to study the HAN architecture.

Ferry's Route : The ferry moves in a closed path repeatedly and stops at the same finite number (σ) of predetermined stops in every cycle. The area is divided into σ disjoint subareas and each stop is associated with one of the subareas. A node belongs to that subarea if and only if its signal is strongest at the associated Ferry stop in comparison with the other stops. At each stop, the ferry serves all the nodes located in the associated subarea.

Let $\{Q_1, Q_2, \cdots, Q_\sigma\}$ represent the location of stops of the ferry. For each i let I_i represent the subarea associated with stop i. We assume that BS is located near Q_1. The indexing in this paper is done in a circular manner.

Arrival process : We consider traffic generated at the nodes which we call "uplink", and traffic that arrives to the nodes which is called "downlink". We shall use for both cases the term "arrival". Uplink/Downlink traffic arrives according to an independent marked point processes $\{T_n, M_n\}$, where T_n is the arrival time of the nth point and $M_n = [L_n^u, L_n^d, \eta_n]$ are the corresponding i.i.d. marks:

- T_n is a Poisson point process with parameter λ,
- L_n^u is the source of the uplink (upload to Ferry) while L_n^d is its downlink (download from Ferry) node. Both L_n^u, L_n^d are in Δ. When the BS is involved in data transfer then we either consider L_n^u or L_n^d appropriately to be the BS. This for example occurs whenever the actual source (resp. actual destination) is outside FWLAN.
- η_n is the size of the message. Its distribution can depend upon L_n^u and or L_n^d. It has finite first and second moments everywhere.

The point processes that counts only the uplink traffic (whose source, is in Δ) to BS is Poisson with λ^u. The point processes that counts only downlink arrivals from BS to Δ (in the SAN and HAN architecture) is Poisson with λ^d. We shall use the superscript u or d to denote uplink or downlink.

For the SAN architecture one of the source or destination is BS itself and hence we find it convenient to define explicitly the exogenous uplink (downlink) Poisson point processes $\{T_n^u, M_n^u\}$ with rate λ^u (λ^d), where $M_n^u = [L_n^u, \eta_n^u]$ ($M_n^d = [L_n^d, \eta_n^d]$) are the corresponding marks.

In case of AUN architecture the downlink/uplink traffic to/from nodes can be exogenous (i.e., routed via BS) or can be from the other nodes in Δ. We view both the downlink/uplink traffic as uplink traffic itself with a source and destiny within Δ. Note here that the actual downlink process also starts with upload of data to the Ferry at node (the node will be BS for exogenous downlink) followed by the download of the data to the destiny node. In this case we model the arrivals with Poisson rate λ^u with Marks given by $M_n = [L_n^u, L_n^d, \eta_n^u]$.

[1]Let W be the set of vectors of expected waiting times achieved by a class of design policies. A vector of $w_1 \in W$ obtained by some dominates another vector w_2 if all entries of w_1 are smaller than or equal to those of w_2 with at least one entry being strictly smaller. A minimum vector in W in the Pareto sense is one that is not dominated by any other vector in W.

Radio channel conditions and service time : The Ferry uses a wireless link to serve the customers. It can receive/transmit the messages from/to the nodes at a distance of d from it at a rate $r(d)$ for some decreasing function $r(.)$. We shall write r^u and r^d for the uplink and downlink rate functions (in case they are different). The upload/download service time of a message with source located at $l^u \in I_i$ close to stop i and sink located at $l^d \in I_j$ is its size divided by the service rate (which depends on the distance between the node's location l^u/l^d and the corresponding Ferry's location Q_i/Q_j) (for example in case of AUN architecture) :

$$B^u(l^u, l^d) = \frac{\eta^u(l^u, l^d)}{r^u(||Q_i - l^u||)} \text{ for uplink,} \quad (1)$$
$$B^d(l^u, l^d) = \frac{\eta^d(l^u, l^d)}{r^d(||Q_j - l^d||)} \text{ for downlink.}$$

Throughout the paper $||.||$ represents either the area (length) of the two (one) dimensional region and or the distance between two points.

Remarks :

- For the special case of routing the data within Δ (AUN architecture), i.e. for data transfer from $l^u \in \Delta$ to $l^d \in \Delta$, the transfer is modeled as two (uplink and downlink) arrivals however with same file sizes, i.e., $\eta^u = \eta^d$ (more details in Section V).
- The service times in SAN architecture depends either only on L^u (for uplink) or only on L^d (for downlink). Hence in this case η^m, B^m depend only on l^m (for $m = u, d$).

The first two moments of the service time at the uplink and downlink queues at station j are b_j^m, $b_j^{(2m)}$ for $m = u, d$ with

$$b_j^m := E\left[B^m(L^u, L^d) | L^m \in I_j\right], \quad (2)$$
$$b_j^{(2m)} := E\left[(B^m(L^u, L^d))^2 | L^m \in I_j\right].$$

(Note that up and downlink service times are defined for all three architectures, where downlink means from the Ferry and uplink - to the Ferry.)

Walking times : After serving all the nodes in a stop Q_i, the ferry walks to the next stop Q_{i+1}. The walking time is $c_1||Q_i - Q_{i+1}|| + c_2$, for some appropriate constants c_1, c_2. The constant c_2 represents the cost for acceleration/deceleration while c_1 represents the cost of speed of the ferry.

The larger the number of stops, the larger the cost of walking will be. Yet at the same time, the lesser the number of stops the cost of serving the nodes increases as then the ferry will have to serve nodes at a more distant points. Because of *these two contrasting costs, one needs to design optimal number of stops for the ferry.*

Pareto Optimality: Let the uplink be handled at σ^u stops and the downlink at σ^d stops. Let ζ^u and ζ^d be two vectors of dimensions σ^u and σ^d respectively whose entries are all strictly positive. Consider the problem of minimizing $\sum_i^{\sigma}(\zeta_i^u E[W_i^u] + \zeta_i^d E[W_i^d])$ where W_i is the waiting time at stop i (the superscript u and d are for uplink and downlink, respectively. Then a policy u that minimizes this sum is clearly Pareto optimal for the problem of minimizing the expected waiting times at all stops.

In the following sections, we model these (uplink/downlink) stops by queues and FWLAN by polling system to compute the objective function of the above para using the existing theory of polling systems.

We now proceed to choose appropriate ζ^u, ζ^d for the objective function, $\sum_i^{\sigma}(\zeta_i^u E[W_i^u] + \zeta_i^d E[W_i^d])$. For any system design policy (design of routes and of number of stops) we can compute the expected waiting times at each queue (see Section VII) using a set of linear equations (see for example [1] for the existence and uniqueness of the solution). However, for one particular choice of the parameter, we can compute explicitly the objective function without computing first the expected waiting time at each queue. This is the case when the objective is to minimize the expected virtual workload[2] in the system, which we shall compute in the following sections using the Pseudo Conservation Laws [7], [5].

III. Sensor Access Network Architecture

Traffic is either uplink from the nodes (sensors) to the BS (through which it can be further routed to the Internet) or downlink from the Internet to nodes of the FWLAN again passing through the BS (which serves as a gateway). The cyclic route of the ferry starts when reaching the BS. It first deposits the (uplink) data collected from all the nodes of Δ in the previous cycle to the BS. It then collects all the downlink data from the BS before walking on to the first stop. The radio connection between the ferry and BS are assumed to be very good and hence one can neglect the time taken by the ferry for serving the BS. While traversing through the path, at every stop i, the ferry first downloads all the data collected from the BS destined to the nodes located in area I_i and then collects all the uplink data messages that have arrived since its last visit. It continues collecting the messages till there are no more uplink data messages in I_i.

A. Polling model and pseudo conservation laws

We model the SAN-FWLAN as a polling system with 2σ queues and analyze its performance using the theory of poling systems. Any polling system consists of a number of queues served by a single server in a cyclic/periodic manner. Various types of polling systems are studied in literature: each differing from the other either in terms of the order in which the server serves the queues or in the service policies at each queue or in the number (finite, countably infinite or continuum) of queues etc. Below we show how SAN-FWLAN can be mapped to a particular polling system and also discuss the exact configuration of the polling system that models it.

Stop Q_i modeled as two $M/G/1$ queues : As a first step, we model each stop, Q_i, as 2 independent queues, the first

[2] **Virtual Workload :** The total workload required by the waiting customers ([6]).

one numbered $2i-1$ is for downlink while the second one numbered $2i$ is for uplink. Downlink and uplink queues have Poisson arrivals with rates λ_i^d and λ_i^u respectively. As the ferry while at stop i, serves all the active nodes in I_i, the effective service times of the queues are given by those $B^u(l^u)$, $B^d(l^d)$ (2) whose l^u/l^d are in I_i. Hence the first two moments of the service times for the upload (download) queue at i are respectively given by $b_i^u, b_i^{(2u)}$ ($b_i^u, b_i^{(2u)}$) of (2). Hence, each stop is modeled as two independent $M/G/1$ queues.

Thus there are 2σ queues served cyclically by the ferry, which represents the server of the polling system.

Service policies at the queues : At each stop Q_i, the uplink queue is served after the downlink queue and since all the uplink messages are collected till there is no more data message left in I_i, the uplink queue is empty when the ferry leaves the stop Q_i. Thus, each of the uplink queues are served with Exhaustive Gating policy. On the other hand, the messages that are transmitted downlink are those present at BS when the Ferry arrives at the BS. In case there were only downlink transmission, this would correspond to the globally gated service discipline [7], [5]. In presence of nodes transmitting (exhaustively) uplink, globally gating is applied only to part of the queues (the downlink).

Walking times : Let V_i represent the time taken by the ferry for walking between queues $i, i+1$ of the polling system. The walking time between the two queues corresponding to the same stop is zero. Thus, these walking times are given by:

$$V_{2i} = c_1||Q_i - Q_{i+1}|| + c_2, \quad V_{2i-1} = 0 \text{ for all } i. \quad (3)$$

We thus have 2σ poling system with half the queues experiencing Exhaustive service while the remaining half receive the globally gated service. With $m = u$ or d, define (for $i = 1, 2, \cdots \sigma$)

$$\begin{array}{llllll} l_j^m & := & Prob\{L_0^m \in I_j\} & \lambda_i^m & := & l_i^m \lambda^m \\ \bar{V}_i & := & \sum_{j<i} V_{2j} & \rho_i^m & := & \lambda_i^m b_i^m \\ \lambda_{2i} & := & \lambda_i^u & \lambda_{2i-1} & := & \lambda_i^d \\ b_{2i} & := & b_i^d & b_{2i-1} & := & b_i^u \\ b_{2i}^{(2)} & := & b_i^{(2d)} & b_{2i-1}^{(2)} & := & b_i^{(2u)} \\ \rho^m & := & \sum_{i=1}^{\sigma} \rho_i^m & \rho & := & \rho^u + \rho^d \\ \bar{V} & := & \bar{V}_{\sigma+1} = c_1 \sum_{i=1}^{\sigma} |Q_i - Q_{i-1}| + \sigma c_2. & & & \end{array}$$

B. Stability

The total load ρ is given by,

$$\begin{aligned} \rho &= \lambda^u \sum_{i=1}^{\sigma} Prob(L^u \in I_i) E\left[\left. \frac{\eta^u(L^u)}{r^u(|Q_i - L^u|)} \right| L^u \in I_i \right] \\ &\quad + \lambda^d \sum_{i=1}^{\sigma} Prob(L^d \in I_i) E\left[\left. \frac{\eta^d(L^d)}{r^d(|Q_i - L^d|)} \right| L^d \in I_i \right] \\ &= \sum_{m=u,d} \lambda^m E\left[\frac{\eta^m(L^m)}{r^m(\min_{1\le i\le\sigma} |Q_i - L^m|)} \right] \end{aligned} \quad (4)$$

A straightforward adaptation of [9], [2], [8] yields the following:

Lemma 3.1: FWLAN is stable if and only if $\rho < 1$. Further ρ is a non increasing function of σ and hence the stability of the system improves as the number of stops, increases. ⋄

From (4) a necessary condition for stability is

$$\sum_{m=u,d} \lambda^m E\left[\frac{\eta^m(L^m)}{\max_{l\in\Delta} r^m(l)} \right] < 1. \quad (5)$$

Note: the fading nature of the wireless medium and the shadowing (which we have not considered in this paper) may further reduce the stability region. Both of them can be introduced to our problem; the way to model up and downlink communications through polling systems remains the same, but the service time distributions become more complex.

For any pair of arrival rates (λ^u, λ^d), the stability condition can be ensured as follows. First, ensure the stability condition for the polling system in which all nodes are located at distance zero from the stations (see (5)). The system now behaves like a wire line system. We request that $\rho < 1$ for this system. If this holds, then by using a sufficient amount of transmission power one can ensure that the stability condition also holds for the wireless system. Alternatively, one can also ensure stability condition by also increasing the number of stops.

C. Virtual Workload and Pseudo conservation laws

As mentioned in the previous section, for Pareto optimal solution, we aim to minimize the expected weighted waiting time $WWT := \sum_i^{\sigma} (\rho_i^u E[W]_i^u + \rho_i^d E[W]_i^d)$, which by virtue of Little's law and Wald's lemma equals the expected virtual workload ([7]) in the system. Our aim is to design a ferry route, more specially chose the number of stops for the ferry so as to minimize the expected virtual workload in the system. By applying the Pseudo Conservation Laws of [7], [5] to our polling model, we get:

$$\begin{aligned} \sum_i^{\sigma} (\rho_i^u E[W]_i^u + \rho_i^d E[W]_i^d) &= \rho \frac{\sum_{j=1}^{2\sigma} \lambda_j b_j^{(2)}}{2(1-\rho)} + \rho \frac{\bar{V}}{2} \\ + \frac{\bar{V}}{2(1-\rho)} \left[\rho^2 - \sum_{i=1}^{\sigma} (\rho_i^u)^2 \right] &+ \rho \frac{\bar{V}(1+\rho)}{2(1-\rho)} + \sum_i^{\sigma} \rho_i^d \bar{V}_i. \end{aligned} \quad (6)$$

The first term in RHS of (6) is always decreasing in ρ, while the rest of the terms are product of a decreasing and increasing terms. The term $\bar{V}$ is increasing at least linearly in σ and hence will eventually dominate the convergence behavior of the virtual workload under more realistic condition of $\lim_{d\to 0} 1/r^m(d) > 0$, $m = u$ or d. Thus the virtual workload initially may decrease with σ because of ρ terms but will eventually increase to infinity if $\lim_{d\to 0} 1/r^m(d) > 0$. Hence, there exist optimal number of stops σ^* for which the virtual workload is minimized.

Symmetric conditions: Assume full symmetry, i.e. that L^u and L^d are uniformly distributed over Δ, the service times independent of the location, the stops at equal distances etc. Further assume that the parameters of uplink and downlink to be same. Then for all i (assuming Q_1 is at the zero location),

$$\begin{aligned} l_i^u &= l_i^d = \frac{1}{\sigma}, \qquad \lambda_i^u = \lambda_i^d = \frac{\lambda}{\sigma}, \\ b_i^u &= b_i^d = b_1 =: \eta_b \sigma \int_{l \in I_1} \frac{1}{r(||l||)} \frac{dl}{||\Delta||} \\ b_i^{(2u)} &= b_i^{(2d)} = b_1^{(2)} =: \eta_b^{(2)} \sigma \int_{l \in I_1} \frac{1}{r(||l||)^2} \frac{dl}{||\Delta||}, \\ \rho_i &= \frac{\lambda b_1}{\sigma} \quad \text{and} \quad \rho = 2\sigma\rho_1 = 2\lambda b_1. \end{aligned}$$

In the above, $\eta_b = E[\eta]$ and $\eta_b^{(2)} = E[\eta^2]$.

The WWT (6) under symmetric conditions simplifies to:

$$\begin{aligned} \sum_i^\sigma \rho_i (E[W]_i^u + E[W]_i^d) &= \rho_1 \sum_i \bar{V}_i \\ &+ \frac{\rho_1 \sigma}{2(1-2\rho_1\sigma)} \left(4\lambda b_1^{(2)} + 4\bar{V} + \bar{V}\rho_1\sigma \left(4 - \frac{1}{\sigma}\right)\right) \end{aligned} \tag{7}$$

In the following we consider some interesting examples and obtain further insights into the problem under consideration.

IV. SAN Examples: Circle, Ring and Rectangle

A. *Ferry moving in a One Dimensional Circular Path*

Assume Δ is a circular path. In this case for each i, I_i represents the interval (interval approximating the arc) $I_i = \left[Q_i - \frac{||Q_i - Q_{i-1}||}{2}, Q_i + \frac{||Q_i - Q_{i+1}||}{2}\right]$ and $\bar{V} = c_1||\Delta|| + \sigma c_2$.

In these examples we consider only the signal attenuation due to path loss. However one can easily incorporate the shadowing and fading effects into our model. Consider that the antenna height difference between the nodes and ferry is one unit. Assuming a path loss coefficient of 2α, for all i,

$$\begin{aligned} r(d) &= \left(\sqrt{1+d^2}\right)^{-2\alpha} = \left(1+d^2\right)^{-\alpha} \qquad (8) \\ b_i &= \eta_b \sigma \int_{-||\Delta||/2\sigma}^{||\Delta||/2\sigma} (1+l^2)^\alpha \frac{dl}{||\Delta||}, \quad \rho_i = \frac{\lambda b_i}{\sigma} \text{ and} \\ b_i^{(2)} &= \eta_b^{(2)} \sigma \int_{-||\Delta||/2\sigma}^{||\Delta||/2\sigma} (1+l^2)^{2\alpha} \frac{dl}{||\Delta||}. \end{aligned}$$

For example for $\alpha = 2$ with appropriate constants,

$$\begin{aligned} b_i &= c_{b1} + \frac{c_{b2}}{\sigma^2} + \frac{c_{b3}}{\sigma^4}, \qquad \rho_i = \frac{c_{\rho 1}}{\sigma} + \frac{c_{\rho 2}}{\sigma^3} + \frac{c_{\rho 3}}{\sigma^5} \text{ and} \\ b_i^{(2)} &= c_{b1}^{(2)} + \frac{c_{b2}^{(2)}}{\sigma^2} + \frac{c_{b3}^{(2)}}{\sigma^4} + \frac{c_{b4}^{(2)}}{\sigma^6} + \frac{c_{b5}^{(2)}}{\sigma^8}. \end{aligned}$$

B. *Annular Ring*

We will assume here that the ferry walks in a circular path C as in the previous example, but serves all the nodes that come in between two concentric rings with the circular path of the ferry placed exactly in the center of the annular ring (Figure 1). Say the rings are separated radially by $2h$ meters. In this case with (R, θ) representing the polar co-ordinates,

$$b_i^m = \frac{1}{l_i^m} \int_{-h}^{h} \int_{\frac{Q_{i,\theta} - Q_{i-1,\theta}}{2}}^{\frac{Q_{i+1,\theta} - Q_{i,\theta}}{2}} \frac{\eta_b^m (Q_i + (R,\theta)) L^m (d\theta dR)}{r^m(||Q_i - [Q_i + (R,\theta)]||)}.$$

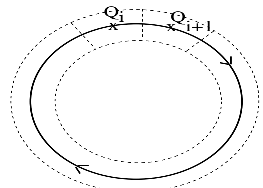

Fig. 1. Circular path of the Ferry moving in a Annular Ring

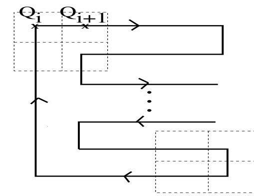

Fig. 2. Zig-Zag path of the Ferry moving in a rectangular area

Under symmetric conditions,

$$b_i = \eta_b \sigma \int_{-h}^{h} \int_{-360/2\sigma}^{360/2\sigma} \frac{1}{r(||Q_1 - [Q_1 + (R,\theta)]||)} \frac{d\theta dR}{||\Delta||}.$$

With fading model (8), approximately (the approximation is good if h is small or when σ is large) for all i,

$$b_i = \eta_b \sigma \int_{-h}^{h} \int_{-||C||/2\sigma}^{||C||/2\sigma} \left(1 + x_1^2 + x_2^2\right)^\alpha \frac{dx_1 dx_2}{||\Delta||}.$$

In Figure 3 we consider a specific example and plot the WWT versus the number of stops. In this example we set, $[c_1, c_2] = [5, 5]$, $h = 0.5$, $||C|| = 16\pi$, $[\eta_b, \eta_1^{(2)}] = [20, 440]$ and $\lambda = 0.01$. We consider two values of α in this example.

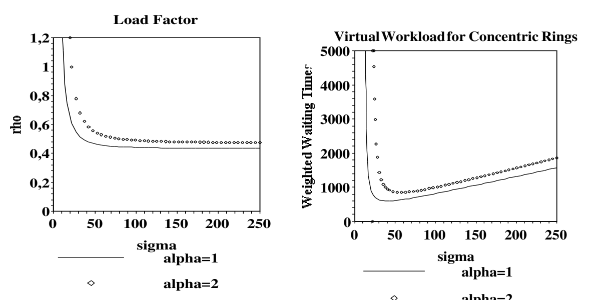

Fig. 3. SAN : Load Factor and Virtual Workload for Annular rings

From the figure, it is seen that ρ decreases with σ, i.e., the stability of the system improves with the increase in the number of stops, for both the values of α, as shown in Lemma 3.1. Further the convergence is faster for smaller value of α. This can be explained in the following way. The number of terms (the polynomial terms in σ) in ρ_1, b_1, $b_1^{(2)}$ increases with α. Hence these terms converge faster to zero with smaller values of α.

As expected and as explained in the previous sections, in Figure 3 the WWT first decreases with σ, reaches minimum at σ^* and then starts to increase to infinity. Also, the optimal number of stops, σ^*, increases with α. The above is intuitively justified, as with smaller values of α, the signal gets attenuated due to fading at a slower rate and hence lesser number of STOPS will suffice.

C. Ferry moving in a Rectangular Area

Here we assume that Δ is a rectangular area of length L and depth H. The ferry moves in a closed zig zag manner as shown in Figure 2. The distance between adjacent stops is same everywhere and the adjacent stops differ from each other either only in horizontal direction or only in vertical direction. At each stop the ferry covers a rectangular area as is shown in the Figure 2 . Hence if $2d$ is the distance between the stops then the number of stops (approximately) will be given by $\frac{LH}{4d^2}$. With the above and with the symmetric conditions,

$$\begin{aligned}\sigma &= \frac{LH}{4d^2} \qquad l_i = \frac{1}{\sigma} \qquad \cdots \\ b_i &= \eta_b \sigma \int_{-d}^{d}\int_{-d}^{d} \left(1 + x_1^2 + x_2^2\right)^\alpha \frac{dx_1 dx_2}{||\Delta||}\end{aligned}$$

For a special case of $\alpha = 2$ this simplifies to (for all i),

$$\begin{aligned}b_i &= c_{b1} + c_{b2}d^2 + c_{b3}d^4 \text{ and similarly} \\ b_i^{(2)} &= c_{b1}^{(2)} + c_{b2}^{(2)}d^2 + c_{b3}^{(2)}d^4 + c_{b4}^{(2)}d^6 + c_{b5}^{(2)}d^8, \\ \rho_i &= c_{\rho 1}d^2 + c_{\rho 2}d^4 + c_{\rho 3}d^6\end{aligned}$$

However in contrast to all the previous models considered, the walking distances increase in a very different way. In this case the walking times are given by (for every $i \leq \sigma$),

$$\begin{aligned}V_{2i} &= 2c_1 d + c_2,\ \bar{V}_i = (2c_1 d + c_2)(i-1) \text{ and} \\ \bar{V} &= \frac{2c_1 LH}{d} + \frac{LHc_2}{d^2}.\end{aligned}$$

Hence the weighted walking time is once again given by (7) with the new $\bar{V}$, $\bar{V}_i$ and ρ_i's.

We next consider the ferry of this section, moving along a zig zag path across a rectangular area in Figure 4. In this example we set, $[c_1, c_2] = [2, 2]$, $LH = 25$, $[\eta_b, \eta_b^{(2)}] = [30, 940]$ and $\lambda = 0.01$. We consider two values of α again. We make similar observations as before and one can support those observations once again as before.

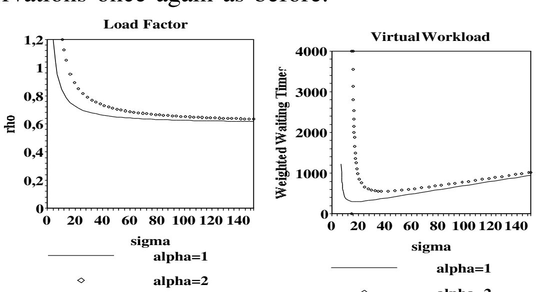

Fig. 4. Load Factor, Virtual Workload for Rectangular area (zig zag path)

V. Autonomous Networks and Extensions

In this section we discuss a more general system with AUN being a special example of it. We consider a system with two BSs. Ferry or the mobile BS works as in AUN system while the other fixed BS acts as a gateway for the external world.

The BS routes only the data flowing out of (or coming from outside) the area Δ while remaining routing, i.e., the routing between nodes inside Δ is done directly by the ferry. Here again, the ferry starts its cycle with BS, which is taken as located close to stop Q_1. One can view the BS as one of the nodes. At every stop i, it first downloads all the data it has for the nodes located in I_i and then uploads the data from the nodes in I_i once again in a exhaustive way.

The new data enters the system only as uplink data. That is, any new data is first uploaded to the ferry when it stops at a nearby stop. The uploaded data is then downloaded to the appropriate node by the ferry itself when the later stops close to destiny node the first time after upload. Note that, the downlink from external world also starts with upload, in this case upload from BS to the ferry.

We shall make heavy use of [16]. Note that the model of [16] requires independent service times. In our case the size of the message that is sent from a source node to a destination service does not change. Therefore the service time up and downlink are naturally correlated. However, if their size is taken to be fixed and not random then one can still use the results of [16]. We thus assume **fixed message sizes** both in this Section as well as in the one on the HAN architecture (in the latter we restrict this assumption to messages that go from one node to another).

A. **Sidi et al., Polling system with rerouting**

We now model this system using the Polling System of Sidi et al., [16], which allows rerouting of the data messages to the polling system after their service.

Each stop is modeled again as two queues. At every stop, the ferry first serves the downlink queue using the usual gated service in contrast to the global gated service of the previous section. This service discipline is more appropriate as in this case the downlink data is generated at all the stops in contrast to the previous section (where the downlink data is generated only at BS). After the downlink queue, the ferry serves the uplink queue using exhaustive service as before. It then walks to the next stop with walking times given by (3). Hence we once again have a 2σ Poling system with half queues experiencing gated service while the rest of the queues experience exhaustive service. However this Poling system is very different from that of the previous section as in this system re routing of the data messages is possible.

In AUN architecture, there are only uplink data arrivals (as external arrivals). Hence $\lambda^d = 0$. The uplink queue at stop Q_i has Poisson arrivals at rate $\lambda_i^u = \lambda^u Prob(L^u \in I_i)$ while the downlink queue at Q_i has arrivals only due to rerouting of data messages. Using the models of Section II, the moments of the work load process/service times at the i^{th} queue of 2σ-Polling system modeling FWLAN will be given by,

$$\begin{aligned}b_{2i} = b_i^u &:= E\left[\frac{\eta^u(L^u, L^d)}{r^u(||Q_i - L^u||)} \middle| L^u \in I_i \right] \\ b_{2i-1} = b_i^d &:= E\left[\frac{\eta^u(L^u, L^d)}{r^d(||Q_i - L^d||)} \middle| L^d \in I_i \right].\end{aligned}$$

The second moments $b_{2i}^{(2)} = b_i^{(2u)}$, $b_{2i-1}^{(2)} = b_i^{(2d)}$ are defined in a similar way.

Every data message is first served (uploaded to the ferry) at a uplink queue. After the first (uplink) service, the messages are routed to a downlink queue in the system according to the probability distribution[3] governing L^d. After the second (downlink) service (download from ferry to destiny node) the message always leaves the system. Thus the probabilities describing the rerouting of the messages in our polling system is given by,

$$\begin{aligned}
P_{2m,2m'-1} &= Prob(L^d \in I'_m) = l^d_{m'} \\
P_{m'',2m} &= 0 \\
P_{2m-1,m''} &= 0 \quad \text{for } 1 \le m, m' \le \sigma,\ 1 \le m'' \le 2\sigma.
\end{aligned}$$

The probabilities of data messages leaving the system at various queues equal:

$$P_{2m,0} = 0 \text{ and } P_{2m-1,0} = 1.$$

The total arrival rates $\{\gamma_i; i \le 2\sigma\}$ (equation (2.1) of [16]), total service rate moments $\{\tilde{b}_i\}$, $\{\tilde{b}_i^{(2)}\}$(equations (2.2), (2.3) of [16]) for FWLAN can easily be computed as :

Downlink	Uplink
$\lambda_{2i-1} = 0$	$\lambda_{2i} = \lambda_i^u$
$\gamma_{2i-1} = l_i^d \sum_{j=1}^{\sigma} \lambda_j^u = l_i^d \lambda^u$	$\gamma_{2i} = \lambda_i^u$
$\tilde{b}_{2i-1} = b_i^d,$	$\tilde{b}_{2i} = b_i^u + \sum_{j=1}^{\sigma} l_j^d b_j^d$
$\tilde{b}_{2i-1}^{(2)} = b_i^{(2d)}$	$\tilde{b}_{2i}^{(2)} = b_i^{(2u)} + \sum_{j=1}^{\sigma} l_j^d b_j^{(2d)}$

B. Stability

With $\rho_i = \gamma_i b_i$ for $i \le 2\sigma$, $\rho = \sum_{i=1}^{2\sigma} \rho_i < 1$ is the stability condition. As before, the load factor is given by:

$$\rho = \lambda^u \sum_{m=u,d} E\left[\frac{\eta^u(L^u, L^d)}{r^m(\min_{1\le i\le\sigma} ||Q_i - L^m||)}\right].$$

Hence as in SAN architecture the stability region improves by increasing the number of stops.

C. Virtual Workload

The virtual workload for this FWLAN can be obtained using equations (6.3), (6.4), (6.5) of [16]. Since FWLAN is modeled by a mixed service polling system as in equation (3.22) [6], one need to add the terms of gated service (given by (6.4), [16]) and exhaustive service ((6.5), [16]) appropriately to obtain the required expression. The gated service ((6.4),[16]) has extra terms in comparison with the exhaustive service ((6.5), [16]). However these extra terms are zero for FWLAN because :

- No external arrivals occur at a downlink/gated service queue ($\lambda_{2i-1} = 0$ for all i). Hence the extra new workload that would have joined the gated service queue, during its own service time, is also zero.
- No data messages are rerouted after a downlink queue and hence $P_{2i-1,j} = 0$ for all i, j. Hence the extra rerouted workload added at a gated service queue (in comparison with exhaustive service queue), due to rerouting to itself, is also zero.

[3]This distribution can be continuous random variable over Δ but probably should have a point mass at the location of the BS to emphasize that BS is the gateway for routing all the data to outside world.

The WWT after all the possible simplifications :

$$\begin{aligned}
&\sum_{i=1}^{\sigma}\left(\lambda_i^u \tilde{b}_{2i} E[W_i^u] + \gamma_{2i-1} b_i^d E[W_i^d]\right) \\
&\quad = \frac{\sum_i^{\sigma} \lambda_{2i}\tilde{b}_{2i}^{(2)}}{2(1-\rho)} - \sum_{i=1}^{2\sigma}\gamma_i \frac{b_i^{(2)}}{2} - \sum_{i=1}^{\sigma}\rho_{2i}(\tilde{b}_{2i} - b_{2i}) \\
&\qquad + \rho\frac{\bar{V}}{2} + \frac{1}{1-\rho}\sum_{i=1}^{\sigma} V_{2i} \sum_{j=1, j\ne i}^{\sigma} \lambda_{2j}\tilde{b}_{2j} \sum_{k=2j+1}^{2i} \rho_k \\
&\qquad + \frac{1}{1-\rho}\sum_{i=1}^{\sigma}\sum_{j=1,j\ne i}^{\sigma} \lambda_i^u l_j^d b_j^d \sum_{k=i}^{j-1} V_{2k}.
\end{aligned}$$

Symmetric Conditions: Under symmetric conditions as in previous section for all $i \le \sigma$,

$$\begin{aligned}
&\gamma_{2i} = \gamma_{2i-1} = \frac{\lambda}{\sigma}, \quad \tilde{b}_{2i} = 2b_1, \quad \tilde{b}_{2i-1} = b_1, \\
&\tilde{b}_{2i}^{(2)} = 2b_1^{(2)} + 2b_1^2 \quad \text{and} \quad \tilde{b}_{2i-1}^{(2)} = b_1^{(2)},
\end{aligned}$$

and the WWT becomes,

$$\begin{aligned}
&\sum_{i=1}^{\sigma}\left(\lambda_i^u \tilde{b}_{2i} E[W_i^u] + \gamma_{2i-1} b_i^d E[W_i^d]\right) \\
&\quad = \frac{\lambda\tilde{b}_2^{(2)}}{2(1-2\rho_1\sigma)} - \frac{\lambda}{2}\left(\tilde{b}_1^{(2)} + \tilde{b}_2^{(2)}\right) - \rho_1\sigma b_1 \\
&\qquad + \rho_1\sigma\bar{V} + \frac{\bar{V}(\sigma-1)\lambda 2b_1\rho_1}{(1-2\rho_1\sigma)} + \bar{V}b_1\lambda\frac{\sigma+1}{2\sigma(1-2\rho_1\sigma)}.
\end{aligned}$$

D. An Example : Ferry moving in a rectangular path

We now consider the ferry moving along a zig zag path in a rectangular area as in Section IV-C, but now using AUN architecture in Figure 5. In this example we set, $[c_1, c_2] = [2, 2]$, $LH = 25$, $[\eta_b, \eta_b^{(2)}] = [30, 940]$ and $\lambda = 0.01$. We consider two values of α again. We make similar observations as before and one can support those observations once again as before.

VI. Hybrid Access Network Architecture

We describe in this section how to model this architecture with polling models using again rerouting as in [16].

At each station i we shall use one upload queue per source-destination pair ij and one download queue. There is in addition another queue per each destination at the BS. Packets generated at node i and destined to node j will be modeled as following. They "arrive" at node i according to a Poisson process and queue up in queue ij. When the Ferry arrives at a station that handles station i, then it serves exhaustively all arrivals at queue ij. This service corresponds to uploading the

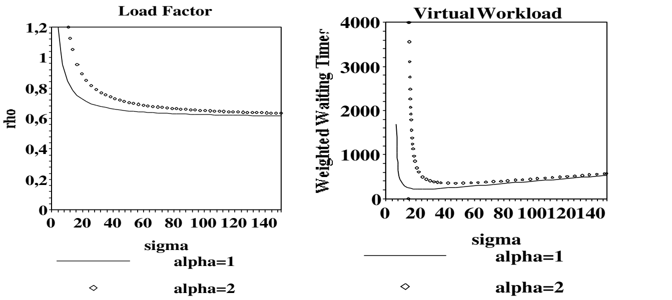

Fig. 5. Load Factor and Virtual Workload for Ferry moving in a Rectangular area with zig zag path for AUN

message to the Ferry. At the end of each service of a message at a queue ij, the message is routed to queue j at the BS and it stays there till the Ferry arrives at the BS.

The time the message from i that was destined to station j waits from the instant it is routed to the BS till the ferry arrives there represents the time that it took for the message to be relayed by the ferry to the BS (as in the real system the routing is not done instantaneously). When the ferry arrives at the BS, all the messages are routed instantaneously to their destinations (defined by the queue in which they wait). Only when the Ferry arrives at the destination queue, the message is served (according to the FIFO order). Again this is not what really happens but it is exactly equivalent to what actually occurs; the reason we use this equivalence is that it allows us to use the tools of Sidi et al [16]. The time that elapses since the message is served at the BS till it is served at the destination node in our model corresponds exactly the time from the instant it is loaded again to the Ferry at the BS till the time it is delivered to the destination.

VII. Conclusions

We have used various uncommon elements of the theory of polling systems to model and analyze the Ferry Based Wireless Local Area Network. Three different architectures have been mapped into tractable polling models. Based on that, we have designed optimal routes and optimal stopping locations taking into account the radio channel considerations. We made a special use of a variant of the globally gated regime and of the rerouting of customers introduced in [16].

We believe that our work can open doors to many other modeling aspects such as adding models to data traffic that generates acknowledgments or to interactive communications; both can be modeled using again ideas from [16].

So far we have focused on the virtual workload in the system. It is possible however to compute also the expected waiting time in each location on the plane, which would then allow to minimize any linear combination of expected waiting times. To that end, we note that when observing the FWLAN at arrival epochs of the Ferry to a station, all the models we have considered are in fact special cases of multi-type branching processes [12] for which expressions for the two first moments (as the unique solutions of a set of linear equations) are available [1]. Since expected waiting time at each station can be expressed in terms of these two moments, one can indeed compute (numerically) the expected waiting time at each queue for a given route of the ferry.

References

[1] Eitan Altman and Dieter Fiems, "Expected waiting time in symmetric polling systems with correlated vacations", Queueing Syst, vol 56, pp 241-253, 2007.

[2] E. Altman, P. Konstantopoulos, and Z. Liu, "Stability, Monotonicity and Invariant Quantities in General Polling Systems", Queuing Syst., 1992, vol. 11, no. 12, pp. 3557.

[3] E. Altman and H. Levy, "Queueing in space", *Advances of Applied Probability* **26**, pp. 1095-1116, 1994.

[4] D. J. Bertsimas and G. Van Ryzin, "A Stochastic and Dynamic Vehicle Routing Problem in the Euclidean Plane", *Operations Research* **39**, No. 4, pp. 601-615, 1991.

[5] O.J. Boxma, "Workloads and waiting times in single-server systems with multiple customer classes", Queuing Systems, 5 (1989) 185-214.

[6] O.J. Boxma, W.P. Groenendijk, "Pseudo-Conservation Laws in Cyclic-Service Systems", Journal of Applied Probability, Vol. 24, No. 4, Dec 1987, 949-964.

[7] O. J Boxma, H. Levy, U Yechiali, "Cyclic Reservation schemes for efficient operation of multiple-queue single-server systems", Annals of Operations Research, 1992, 187-208.

[8] Foss, S.G. and Chernova, N.I., "Comparison Theorems and Ergodic Properties of the Polling Systems," Probl. Peredachi Inf., 1996, no. 4, pp. 46-71.

[9] Georgiadis, L. and Szpankowski, W., "Stability of Token Passing Rings", Queuing Syst., 1992, vol. 11, no. 1-2, pp. 7-33.

[10] A. Khamisy, E. Altman and M. Sidi , "Polling Systems with Synchronization Constraints ", Annals of Operations Research , Vol. 35, special issue on Stochastic Modeling of Telecommunication Systems, Eds. P. Nain and K. W. Ross, pp. 231-267, 1992.

[11] G. Laporte, "What You Should Know about the Vehicle Routing Problem", GERAD, HEC Montreal, G-2007-59, Aug 2007.

[12] J.A.C. Resing. "Polling systems and multi-type branching processes", *Queuing Systems*, 13:409–426, 1993.

[13] W. Saad, Z. Han, T. Basar, M. Debbah and A. Hjorungnes, "A Selfish Approach to Coalition Formation among Unmanned Air Vehicles in Wireless Networks", Gamenets, Istanbul, Turkey, 2009.

[14] Saleh Yousefi, Eitan Altman, Rachid El-Azouzi and Mahmood Fathy, "Connectivity in vehicular ad hoc networks in presence of wireless mobile base-stations", in the Proceedings of the 7th International Conference on ITS Telecommunications, June 6-8, Sophia-Antipolis, France.

[15] Y. Shi and Y. T. Hou, "Theoretical results on base station movement problem for sensor network," in IEEE INFOCOM 08, 2008.

[16] M. Sidi, H. Levy and S. W. Fuhrmann, "A queuing network with a single cyclically roving server", *Queuing Systems* **11**, (special issue on Polling Models, Eds. H. Takagi and O. Boxma), pp.121-144, 1992.

[17] H. Takagi, "*Analysis of Polling Systems*", The MIT Press, 1986.

[18] M. M. B. Tariq, M. Ammar, and E. Zegura, "Message ferry route design for sparse ad hoc networks with mobile nodes", in Proc. of ACM MobiHoc, Florence, Italy, May 22-25, 2006, pp. 37-48.

[19] V. M. Vishnevskii and O. V. Semenova, "Mathematical Methods to Study the Polling Systems", Automation and Remote Control, 2006, Vol. 67, No. 2, pp. 173-220. 2006.