Autumn Semester **December 31, 2021**

IE 605: Engineering Statistics

Tutorial 1

Exercise 1 *Three fair dice are thrown. What is the sample space and event space of this experiment. What is the probability that the same number appears in exactly two of three dice?*

Exercise 2 *Express each of the following events in terms of the events A, B and C as well as the operations of complimenting, union and intersection:*

1. *at least one of the events A, B, C occurs;*
2. *at most one of the events A, B, C occurs;*
3. *none of the events A, B, C occurs;*
4. *all three events A, B, C occur;*
5. *exactly one of the events A, B, C occurs;*
6. *events A and B occur, but not C;*
7. *either event A occurs or, if not, then B also does not occur.*

In each case draw the corresponding Venn diagrams.

Exercise 3 *A coin is tossed until a head appears twice in a row. What is the sample space for this experiment. What is the probability that it will be tossed exactly four times assuming it is a fair coin?*

Exercise 4 *Out of the students in a class, 60% are geniuses, 70% love chocolate, and 40% fall into both categories. Determine the probability that a randomly selected student is neither a genius nor a chocolate lover.*

Exercise 5 *A six-sided die is loaded in a way that each even face is twice as likely as each odd face. Construct a probabilistic model for a single roll of this die, and find the probability that a 1, 2, or 3 will come up.*

Exercise 6 *Let $(\Omega, \mathcal{F}, \mathcal{P})$ denote a probability space. For any $A, B, C \in \mathcal{F}$, prove the following properties.*

- *Let $A \subset B$ show that $P(A) \leq P(B)$.*
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
- $P(A) + P(A^c) = 1$
- $P(\phi) = 0$

Exercise 7 *The conditinal probability of A given B, where $P(B) \neq 0$, is defined by:*

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

Define $P_B(A) = P(A|B)$. Show the $P_B(\cdot)$ is also a probability functions (verify the 3 axioms of probability).

Exercise 8 *Does a set of events being pairwise independent implies that they are independent? If yes, prove it. Otherwise give a counter example.*

Exercise 9 *If $\{A_i i \geq 1\}$ denote a sequence of events. Show that*

$$P(\cup_i^\infty A_i) \leq \textstyle\sum_{i=1}^\infty P(A_i).$$

The above bound is known as union bound.

Exercise 10 *A product is manufactured by two factories A and B. 80% of the product is manufactured in company A and rest in company B. 30% of the product manufactured by company A are defective while 10% of the product manufactured by company B are defective. A sample of the product is randomly selected from the market. Then,*

1. *What is the probability that the sample is defective.*
2. *What is the probability that a defective sample in the market is manufactured at company A.*

Exercise 11 *A particular webserver may be working or not working. If the webserver is not working, any attempt to access it fails. Even if the webserver is working, an attempt to access it can fail due to network congestion beyond the control of the webserver. Suppose that the a priori probability that the server is working is 0.8. Suppose that if the server is working, then each access attempt is successful with probability 0.9, independently of other access attempts. Find the following quantities.*

1. *P(first access attempt fails)*
2. *P(server is working | first access attempt fails)*
3. *P(second access attempt fails | first access attempt fails)*

4. *P(server is working | first and second access attempts fail).*

Exercise 12 *If $E_i, i = 1, 2, \ldots, n$ are a collection of events. Show that* $P(E_1 \cup E_2 \cup ... \cup E_n) = \sum_i P(E_i) - \sum_{i<j} P(E_iE_i) + \sum_{i<j<k} P(E_iE_jE_k) - \sum_{i<j<k<l} P(E_iE_jE_kE_l) + ... + (-1)^{n+1}P(E_1E_2....E_n)$

Exercise 13 *Assume that each child who is born is equally likely to be a boy or a girl.If a family has two children, what is the probability that both are girls given that*

1. *The eldest is a girl*
2. *At least one is a girl*

Exercise 14 *Two dice are rolled. What is the probability that at least one is a six? If the two faces are different, what is the probability that at least one is a six?*

Exercise 15 *Three dice are thrown. What is the probability the same number appears on exactly two of the three dice?*

Exercise 16 *Suppose that 5 percent of men and 1 percent of women are color-blind. A color-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females.*

Exercise 17 *A and B play until one has 2 more points than the other. Assuming that each point is independently won by A with probability p, what is the probability they will play a total of 2n points? What is the probability that A will win?*