Autumn Semester **December 31, 2021**

IE605:Engineering Statistics

Tutorial 3

Exercise 1 *Let X and Y be independent exponential random variables with respective parameters λ_1 and λ_2. Find the distribution of the following.*

- $\min(X, Y)$
- $\max(X, Y)$

Exercise 2 *Let X, Y, Z be discrete random variables. Show the following:*

- $\mathbb{E}(Z) = \mathbb{E}(\mathbb{E}(Z|Y))$
- $\mathbb{E}(Z) = \mathbb{E}(\mathbb{E}(Z|X, Y))$

Exercise 3 *A bag contains 3 white, 6 red and 5 blue balls. A ball is selected at random, it's color is noted and is then replaced in the bag before making the next selection. In all 6 selections are made. Let X = the number of white balls selected and Y = number of blue balls selected. Find $E[X|Y = 3]$.*

Exercise 4 *If X_1 and X_2 are independent binomial random variables with respective parameters (n_1, p) and (n_2, p). Calculate the conditional probability mass function of X_1 given that $X_1 + X_2 = m$.*

Exercise 5 *Give an example of two random variables X and Y that are uncorrelated but not independent.*

Exercise 6 *Suppose X and Y have joint density function $f_{X,Y}(x, y) = c(1 + xy)$ if $2 \leq x \leq 3$ and $1 \leq y \leq 2$, and $f_{X,Y}(x, y) = 0$ otherwise.*

1. *Find c.*
2. *Find f_X and f_Y.*

Exercise 7 *An insurance company supposes that the number of accidents that each of its policyholders will have in a year is Poisson distributed, with the mean of the Poisson depending on the policyholder. If the Poisson mean of a randomly chosen policyholder has a gamma distribution with density function,*

$$g(\lambda) = \lambda e^{-\lambda}, \qquad \lambda \geq 0$$

what is the probability that a randomly chosen policyholder has exactly n accidents next year?

Exercise 8 *Suppose that the number of people who visit a yoga studio each day is a Poisson random variable with mean* λ*. Suppose further that each person who visits is, independently, female with probability p or male with probability* $1 - p$*. Find the joint probability that exactly n women and m men visit the academy today.*

Exercise 9 *A chicken lays* n *eggs. Each egg independently does or doesn't hatch, with probability* p *of hatching. For each egg that hatches, the chick does or doesn't survive (independently of the other eggs), with probability* s *of survival. Let* N *Bin*(n, p) *be the number of eggs which hatch,* X *be the number of chicks which survive, and* Y *be the number of chicks which hatch but don't survive (so* $X + Y = N$*). Find the marginal PMF of* X*.*