Autumn Semester 2020 **December 31, 2021**

IE 605: Engineering Statistics

Tutorial 4

Example 1. *For following moment generating functions, identify the distributions:*

1. $$M_X(u) = \frac{2}{2-u}, \quad \textit{for } u \in (-2, 2)$$

2. $$M_X(u) = \left(\frac{e^u + 2}{3}\right)^{1050}, \quad \textit{for all } u \in \mathbb{R}$$

3. $$M_X(u) = e^{3.5(e^u - 1)}, \quad \textit{for all } u \in \mathbb{R}$$

Example 2. *The joint density of X and Y is given by*

$$f(x,y) = \begin{cases} \frac{1}{2} y e^{-xy}, & 0 < x < \infty, 0 < y < 2 \\ 0, & \textit{otherwise} \end{cases}$$

What is $\mathbb{E}\left[e^{X/2} | Y = 1\right]$*?*

Example 3. *Let* X, Y, Z *be random variables, and* $g : \mathbb{R} \to \mathbb{R}$*. Assuming that the expectations exist, prove:*

1. $E[X|Y] = E[X]$ *if* X *and* Y *are independent.*
2. $E[Xg(Y)|Y] = g(Y)E[X|Y]$*. In particular,* $E[g(Y)|Y] = g(Y)$*.*

Example 4. *Given a communication system, where each data packet consists of 1000 bits. Due to the noise, each bit may be received in error with probability 0.1. It is assumed bit errors occur independently. Using Central Limit Theorem, find the probability that there are more than 120 errors in a certain data packet?*

Example 5. *The lifetime of a special type of battery is a random variable with mean 40 hours and standard deviation 20 hours. A battery is used until it fails, at which point it is replaced by a new one. Assuming a stockpile of 25 such batteries, the lifetimes of which are independent, approximate the probability that over 1100 hours of use can be obtained.*

Example 6. *Let* $X_1, X_2, \ldots, X_{25}$ *be i.i.d. with the following PMF.*

$$P_X(k) = \begin{cases} 0.6 & \textit{if } k = 1, \\ 0.4 & \textit{if} k = -1, \\ 0 & \textit{otherwise} \end{cases}$$

And let $Y = X_1 + X_2 + \ldots + X_{25}$. Using the CLT, estimate $P(3.5 \leq Y \leq 6.5)$.

Example 7. *You have invited 64 guests to a party. You need to make sandwiches for the guests. You believe that a guest might need 0, 1 or 2 sandwiches with probabilities 0.25, 0.5, and 0.25 respectively. You assume that the number of sandwiches each guest needs is independent from other guests. How many sandwiches should you make so that you are 95% sure that there is no shortage?*

Example 8. *There are 100 men on a plane. Let X_i be the weight (in pounds) of the ith man on the plane. Suppose that the X_i's are i.i.d., with mean 170 and standard deviation 30. What is the probability that the total weight of the men on the plane exceeds 18,000 pounds.*

Example 9. *Show that*

$$\lim_{n\to\infty} e^{-n} \sum_{k=0}^{n} \frac{n^k}{k!} = \frac{1}{2}.$$

Example 10. *Let $X \sim Binomial(n, p)$. Using Chebyshev's inequality, find an upper bound on $P(X \geq \theta n)$, where $p < \theta < 1$. Evaluate the bound for $p = 1/2$ and $\theta = 3/4$*

Example 11. *A coin is weighted so that its probability of landing on heads is 20%. Suppose the coin is flipped 20 times. Using Markov's inequality, find a bound for the probability it lands on heads at least 16 times. Compare this bound with actual probability.*

Example 12. *Let X be any RV, and suppose that the MGF of $X, M(t) = \mathbb{E}\left[e^{tX}\right]$, exists for every $t > 0$. Then for any $t > 0$ show that $\mathbb{P}\left\{tX > s^2 + logM(t)\right\} < e^{s^2}$.*