Autumn Semester 2020 **December 31, 2021**

IE 605: Engineering Statistics

Tutorial 5

Example 1. *Let $x_1, x_2, \ldots, x_n$ be any number. Define $\bar{x} = \frac{1}{n}\sum_{i=1}^{n}$ and $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$. Show that*

- $\min_a \sum_i^n (x_i - a)^2 = \sum_i^n (x_i - \bar{x})^2$
- $s^2 = \frac{1}{n-1}\sum_{i=1}^{n} x_i^2 - n\bar{x}$

Example 2. *Let $U \sim \mathcal{N}(0,1)$ and $V \sim \chi^2_{n-1}$, where $n > 1$ is a positive integer and the random variables U and V are independent of each other. Define transforms $X = \frac{U}{\sqrt{V/(n-1)}}$ and $Y = V$.*

- *Find the joint pdf of (U, V)*
- *Find the joint distribution of (X, Y)*
- *Find the marginal distribution of X.*
- *Argue that X has Student's t distribution with $n - 1$ degrees of freedom.*

Example 3. *A manufacturer receives a lot of 100 parts from a vendor. The lot will be unacceptable if more than five of the parts are defective. The manufacturer is going to select randomly K parts from the lot for inspection and the lot will be accepted if no defective parts are found in the sample.*

1. *How large does K have to be to ensure that the probability that the manufacturer accepts an unacceptable lot is less than 0.10?*
2. *Suppose the manufacturer decides to accept the lot if there is at most one defective in the sample. How large does K have to be to ensure that the probability that the manufacturer accepts an unacceptable lot is less than 0.10?*

Example 4. *Let X and Y are $\mathcal{N}(0,1)$ random variables. Define $Z = \min(X, Y)$. Prove that $Z^2 \sim \chi^2_1$.*

Example 5. *Suppose X and Y are independent $N(0,1)$ random variables. Find $P(X^2 + Y^2 < 1)$.*

Example 6. *Show that Beta distribution belong to exponential family of distributions*

Example 7. *Let $X_1, X_2, \ldots, X_n$ is a random sample drawn from a populations $\mathcal{N}(0,1)$. Let $\bar{X}$ and S^2 be the sample mean and sample variance. Show that $\bar{X}$ and S^2 are independent.*

Example 8. *Let* $X \sim Gamma(\alpha, \lambda)$*. Find MGF, first and second moment of* X.

Example 9. *Let* $X_1, X_2, \ldots, X_n$ *are independent with* $X_i \sim \chi^2_{n_i}$ *for all* $i = 1, 2, \ldots, n$*. Let* $p = \sum_{i=i}^{n} n_i$*. Show that* $\sum_{i=1}^{n} X_i \sim \chi^2_p$

Example 10. *Let* $X_1, \ldots, X_n$ *be iid with pdf* $f_X(x)$*. Let* $\bar{X}$ *be the sample mean and* $X = n\bar{X}$*. Show that* $f_{\bar{X}}(x) = nf_X(nx)$.

Example 11. *Let* $\bar{X}_n$*and* S_n^2 *are sample mean and sample variance of random sample* $X_1, X_2, \ldots, X_n$*. Let* X_{n+1}*the sample is obtained. Establish the following recursion relations.*

- $\bar{X}_{n+1} = \frac{X_{n+1} + n\bar{X}_n}{n+1}$
- $nS_{n+1}^2 = (n-1)S_n^2 + \left(\frac{n}{n+1}\right)(X_{n+1} - \bar{X}_n)^2$