Autumn Semester 2020 **December 31, 2021**

IE 605: Engineering Statistics

Tutorial 6

Example 1. *Let* X_1, X_2, X_3, X_4, X_5 *be a sample from* $N(0, 4)$. *Find* $\mathbb{P}\left\{\sum_{i=1}^{5} X_i^2 \geq 5.75\right\}$.

Example 2. *Let* $X_1, X_2, \ldots, X_n$ *be a random sample from* $Poisson(\lambda)$. *Find* $var(S^2)$*, and compare it with* $var(\bar{X})$.

Example 3. *Show that for a random sample of size 2 from* $\mathcal{N}(0, \sigma^2)$ *population,* $\mathbb{E}\left[(X_{(1)}\right] = \frac{-\sigma}{\sqrt{\pi}}$.

Example 4. *Let* X*,* Y *and* Z *be independent uniform random variables on the interval* $(0, a)$. *Let* $W = \min\{X, Y, Z\}$. *What is the expected value of* $\left(1 - \frac{W}{a}\right)^2$

Example 5. *Let*$X_1, X_2, ..., X_n$ *be a random sample from a population* X *with uniform distribution on the interval* $[0, 1]$. *What is the probability distribution of the sample range* $W := X_{(n)} - X_{(1)}$*?*

Example 6. *Let* X *be a random variable with an* $F_{p,q}$ *distribution.*

- *Derive the pdf of* X.
- *Derive the mean and variance of* X.
- *Show that* $\frac{\frac{p}{q}X}{1+\frac{p}{q}X}$ *has a beta distribution with parameters* $p/2$ *and* $q/2$.

Example 7. *Show that in odd samples of size* n *from* $Unif(O, 1)$ *population, the mean and variance of the distribution of median are* $1/2$ *and* $\frac{1}{4(n+2)}$ *respectively, i.e., find the expectation and variance of the median* $X_{(m+1)}$ *where* $n = 2m + 1$.

Example 8. *Show that sample standard deviation is not unbiased, but is consistent.*

Example 9. *Let* $U_i, i = 1, 2, \ldots$ *, be independent uniform*$(0, 1)$ *random variables, and let* X *have distribution*

$$P(X = x) = \frac{c}{x!}, \quad x = 1, 2, 3, ...,$$

where $c = \frac{1}{e-1}$. *Find the distribution of*

$$Z = \min\{U_1, ..., U_X\}.$$

Example 10. *Let* $X_i, i = 1, 2, 3$ *, be independent with n(i,*i^2*) distribution. For each of the following situations, use the* X_i *to construct a statistic with the indicated distribution*

1. *chi squared with 3 degrees of freedom*
2. *t distribution with 2 degrees of freedom*
3. *F distribution with 1 and 2 degrees of freedom*

Example 11. *Let* $X_1, X_2, \ldots, X_n$ *be a random sample from a population with pdf.*

$$f(x) = (1/\theta) \quad \textit{for all } 0 \le x \le \theta \tag{1}$$

Let $X_1 < X_2 \cdots < X_n$ *be the order statistics. Show that* X_1/X_n *and* X_n *are independent random variables.*

Example 12. *Discuss how can you generate* $F_{p,q}$ *distribution from Uniform random variables. Assume* p, q *are integers.*