Autumn Semester 2021 **December 31, 2021**

IE 605: Engineering Statistics

Tutorial 7

Example 1. *Consider generating random number from the following distributions starting by generating a random number from $U \sim Uniform(0,1)$:*

- *Show that both $-logU$ and $-log(1-U)$ are exponential random variables*
- *Show that $X = \frac{U}{1-U}$ is a logistic(0,1) random variable.*

Note: *You can find PDF and more about the logistic distribution* ***here***

Example 2. *The Box-Muller method for generating normal pseudo-random variables is based on the transformation*

$$X_1 = cos(2\pi U_1)\sqrt{-2log(U_2)}, \qquad X_2 = sin(2\pi U_1)\sqrt{-2log(U_2)}$$

where U_1 and U_2 are iid $Uniform(0,1)$. Prove that X_1 and X_2 are independent $Normal(0,1)$ random variables.

Example 3. *Park et.al. (1996) describe a method for generating correlated binary variables based on the following scheme:*

Let X_1, X_2, X_3 be independent Poisson random variables with mean $\lambda_1, \lambda_2, \lambda_3$ respectively, and create the random variables

$$Y_1 = X_1 + X_3 \qquad \text{and} \qquad Y_2 = X_2 + X_3.$$

1. *Show that $Cov(Y_1, Y_2) = \lambda_3$.*
2. *Define $Z_i = \mathbb{I}(Y_i = 0)$ and $p_i = e^{-(\lambda_i + \lambda_3)}$. Show that Z_i are $Bernoulli(p_i)$ with*
$$Corr(Z_1, Z_2) = \frac{p_1 p_2 (e^{\lambda_3} - 1)}{\sqrt{p_1(1-p_1)}\sqrt{p_2(1-p_2)}}.$$

Example 4. *Let us propose an algorithm to generate a random variable $Y \sim Beta(a,b)$:*

- ***Step 1:*** *Generate (U, V), U and V are independent $Uniform(0,1)$*
- ***Step 2:*** *If $U < \frac{1}{c} f_Y(V)$ where $c \geq max_y f_Y(y)$, set $Y = V$; otherwise return to Step 1.*

Does the above algorithm actually generate a $Beta(a,b)$ random variable? If yes, prove. If no, why? (Hint: Try to check $P(Y \leq y)$ and write in terms of something of the form $P(V|U)$)

Example 5. *Solve the following:*

1. *Suppose it is desired to generate* $Y \sim Beta(a, b)$*, where* a *and* b *are not integers. Show that using* $V \sim Beta([a], [b])$ *will result in a finite value of* $M = \sup_y \frac{f_Y(y)}{f_V(y)}$.
2. *Suppose it is desired to generate* $Y \sim Gamma(a, b)$*, where* a *and* b *are not integers. Show that using* $V \sim Gamma([a], b)$ *will result in a finite value of* $M = \sup_y \frac{f_Y(y)}{f_V(y)}$.
3. *Show that, in each of parts (1) and (2), if* V *had parameter* $[a] + 1$*, then* M *would be infinite.*