Autumn Semester 2021 **December 31, 2021**

IE 605: Engineering Statistics

Tutorial 11

Exercise 1. *1. If $L(x)$ and $U(x)$ satisfy $\mathbb{P}\{L(X) \leq \theta\} = 1 - \alpha_1$ and $\mathbb{P}\{U(X) \geq \theta\} = 1 - \alpha_2$, and $L(x) \leq U(x)$ for all x, show that $\mathbb{P}\{L(X) \leq \theta \leq U(X)\} = 1 - \alpha_1 - \alpha_2$.*

2. *If T is a continuous random variable with cdf $F_T(t|\theta)$ and $\alpha_1 + \alpha_2 = \alpha$, show that an α level of acceptance region of the hypothesis $H_0 : \theta = \theta_0$ is $\{t : \alpha_1 \leq F_T(t|\theta_0) \leq 1 - \alpha_2\}$, with associated confidence $(1 - \alpha)$ set $\{\theta : \alpha_1 \leq F_T(t|\theta) \leq 1 - \alpha_2\}$.*

Exercise 2. *Let $X_1, \ldots, X_n$ be a random sample from a $N(0, \sigma_X^2)$ and $Y_1, \ldots, Y_n$ be a random sample from a $N(0, \sigma_Y^2)$, independent of the $X's$. Define $\lambda = \frac{\sigma_Y^2}{\sigma_X^2}$.*

1. *Find the level α LRT of $H_0 : \lambda = \lambda_0$ vs $H_1 : \lambda \neq \lambda_0$.*

2. *Express the rejection region of the LRT of part (a) in terms of F-distributed random variable.*

3. *Find a $(1 - \alpha)$ confidence interval for λ.*

Exercise 3. *Let $\bar{X}$ be the mean of a random sample of size n from $N(\mu, 16)$. Find the smallest sample size n such that $(\bar{X} - 1, \bar{X} + 1)$ is a 0.90 level confidence interval for μ.*

Exercise 4. *Under the one-way ANOVA assumptions:*

1. *Show that the set of statistics $(\bar{Y}_1, \bar{Y}_2, \ldots, \bar{Y}_k, S_p^2)$ is sufficient for $(\theta_1, \theta_2, \ldots, \theta_k, \sigma^2)$.*

2. *Show that $S_p^2 = \frac{1}{N-k} \sum_{i=1}^{k} (n_i - 1) S_i^2$ is independent of each $\bar{Y}_i, i = 1, 2 \ldots, k$.*

Exercise 5. *Show how to construct a t test for*

1. *$H_0 : \sum_i a_i\theta_i = \delta$ vs $H_1 : \sum_i a_i\theta_i \neq \delta$.*

2. *$H_0 : \sum_i a_i\theta_i \leq \delta$ vs $H_1 : \sum_i a_i\theta_i > \delta$, where δ is a specified constant.*

Exercise 6. *For any set of constants $a = (a_1, \ldots, a_k)$ and $b = (b_1, \ldots, d_k)$, show that under one-way ANOVA assumptions,*

$$cov(\sum_i a_i\bar{Y}_i, \sum_i b_i\bar{Y}_i) = \sigma^2 \sum_i \frac{a_i b_i}{n_i}.$$