Fixed Life Perishable Inventory Problem and Approximation under Price Promotion

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Abstract

The problem considered here is to determine an optimal ordering policy and price promotion decision in each period, for a perishable item over a finite horizon. The fixed life perishability problem (FLPP) requires an age-wise profile of inventory. As the life of the item increases, the size of the state space also increases. It may be possible to reduce the size of a state space by collapsing some of its elements, without much loss of information. In this paper, we propose an approximation to group some of fresher inventory elements of a state space. By an extensive numerical analysis, we conclude that for items having life more than 4 periods, an approximation of grouping the elements of an inventory vector representing inventory of life more than 4 period into the last element of the state space is resonable. The behavior of the reward function is analyzed and shown that the reward function is concave in ordering quantity. The error in average reward for approximation model is less than 4%.

1 Introduction

Controlling inventories of perishable items poses a significant challenge due to limited useful life of items. These items if not used before the expiry date would outdate and there would be an additional cost of outdating of perished items. To maximize total reward over a finite horizon, price promotions can be used to clear off the sale of items having less remaining useful life. In a price sensitive market price promotion can be a reasonable option to stimulate the demand. Problem considered here is to determine an optimal time to announce price promotion and optimal ordering quantity in each period for a perishable item over a finite horizon. It is assumed that after the fixed horizon the product has to be withdrawn from the market. This is a quite realistic assumption; as due technological advancement or to be competitive in market, management may withdrew the old products and introduce new ones.

There is only one markdown regarding to price promotion is allowed in the model, as there can be additional costs associated with the withdrawl of the promotion. For the problem considered, an age-wise profile of inventory should be known explicitly in each period. Various costs related to ordering quantity, inventory holding, penalty cost for lost sales, disposal cost for outdated items. There is a fixed cost related to price promotion in each period if product is price promoted. A stochastic dynamic programming approach is used to maximize the expected reward over a finite horizon.

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Most of the times, the optimal policies are either too difficult to compute or to implement. Therefore, it may be possible to develop an approximation approach which reduces the computations and gives solution near to the optimal, within permissible limits. For perishable inventory control, the state space comprises of age-wise profile of items. As life of item increases, the size of state space also increases. In this context, we propose an approximation to reduce the state space by collapsing some of the elements of inventory vector. A numerical analysis is carried out to study the behavior of optimal policies with approximation approach, by varying various parameters.

The paper is organized as follows. Next section reviews the literature on determination of optimal ordering polices and price promotion decisions for a perishable item. In section 3, a mathematical model for the problem considered is presented. A numerical analysis and observations made are also presented. Section 4 proposes an approximation approach for the problem considered. A detailed numerical analysis considering the approximation approach is presented.

2 Literature Review

Perishability refers to decrease in value or usability of product over time due to the inherent characteristics of product; whereas obsolescence refers to loss in value of product due external factors such as, technological innovations, new product introduction by competitor, *etc.* The literature on perishable inventory to determine optimal ordering policies, considered different scenarios related to demand patterns, issuing policies, review periods of inventory, *etc.*

Nahmias [8] and Fries [6] have considered problem of determining optimal policies for items with fixed lifetime. Nahmias considered the case where unsatisfied demand is fulfilled by backlogging, while backlogging is not allowed in Fries' model. A recursive dynamic programming approach is used to obtain minimum expected discounted cost function. In Fries' model, the optimal policy for lifetime more than two periods and length of horizon greater than or equal to two periods is of the form: Order up-to a level whenever total inventory level drops below a certain level, *i.e.* (s, S) policy. The decision to order or not depends on whether or not the total inventory is less than a critical number, and it does not depends on the age distribution of inventory or on the number of periods till the end horizon. However, the decision how much to order depends on both the age distribution and the number of periods till the end horizon. Cohen [3] has computed stationary distribution of total inventory for two period lifetime problem and thereby derived the optimal critical number policy.

The literature on dynamic pricing of perishable products considered different cases of pricing of items like, single markdown over a fixed horizon, multiple price promotions, price decreases with time, *etc.* Abad [1] has considered the problem of dynamic pricing and lot-sizing with backordering with known demand. The problem is similar to yield management problem that is observed in the airline and hotel industries; where one generally deals with fixed time, fixed stock and dynamic pricing. However, FLPP differs in that as it considers fixed time, dynamic pricing and variable stock. Cheng and Sethi

[4] proposes a special structure policy (S^0, S^1, P) where, S^0 is order-up-to level when product is not promoted and initial inventory is $x < S^0$, and S^1 is order-up-to level when product is promoted and P is the level above which the product is promoted.

Most of the literature on perishable inventory considered the problem as a multi-dimensional dynamic program; the dimension being equal to the lifetime of the product. Hence, the computations required increases as the state space. Therefore, it might worth to develop some good approximations to reduce the efforts and complexities. Nahmias [10] has proposed an approximation to reduce the state space. The approximation suggests that an inventory vector $(x_{L-1}, x_{L-2}, \ldots, x_1)$ may be collapsed into the vector $(x_{L-1}, \ldots, x_{k+1}, \sum_{j=1}^{k} x_j)$ without excessive loss of information. Here, x_i is number of units having remaining life of i periods. The collapsed vector will have dimension $L - k$. The approximation is based on the observation that the optimal ordering quantity is more sensitive to changes in fresher inventory than older inventory. For example, if on-hand older inventory stock increases by 1 unit then the ordering quantity reduces, but by less than 1 unit. In an another paper, Nahmias used the approach of bounding the expected deterioration cost and an approximate transfer function is used to calculate the critical number policy.

Goyal and Giri [2], have surveyed the literature on the modeling of deteriorating items.

3 Konda' Model

Konda' model is to determine a possible price promotion decision and optimal ordering quantity in each period for a perishable item which is to be withdrawn from the market after a fixed number of periods. It is assumed that once price of item is promoted, it continues till the end of horizon. This is an assumption and different possibilities of price promotions can be easily modeled in this framework. There is a fixed promotion cost for the promoted states to consider costs related to advertising, packaging, *etc.* Demand distributions of both promotion and no-promotion cases are known and identical in each period. It is assumed that, demand distribution in promoted case is stochastically dominant than that of no promoted case. First-in-First-out rule (FIFO) is used as issuing policy. It is assumed that usability of the items remains same during entire lifetime.

A similar problem of announcing price promotion is also modeled by Dave [5] using a Markov Decision process approach.

Let, $r(s_t, (a, y_t))$ is the one period expected reward in the t^{th} stage for the action (a, y_t) in state s_t; the state s_t is represented by $(x_{1t}, x_{2t}, \ldots, x_{L-1t})$ which is the inventory vector at the beginning of period t, where, x_{it} is the number of units in period t having remaining shelf life of i periods, $i = 1, ..., L - 1$ and $t = 0, ..., N$, where N is length of planning horizon. The value of a is either p (if promotion is offered) or n (if promotion is not offered). y_t is the ordering quantity in any period t. D denotes the maximum demand, u_{ta} the random variable of demand in period t with known distribution when decision a is made, I the maximum inventory and i_t the total inventory on hand at the beginning of period t, and j_t the initial inventory on-hand which is transferred from period $t - 1$

to t. The fixed cost of promotion is K which is incurred in the periods in which product is promoted.

Inventory capacity constraint is,

$$\sum_{i=1}^{i=L} x_{it} \leq I \quad \forall s_t \in S \ . \tag{1}$$

The following are the inventory balance equation,

$$j_{t+1} = i_t - \mu_t - [0, (x_{1t} - \mu(\cdot))]^+, \tag{2}$$

$$i_t = y_t + j_t = \sum_{i=1}^{L} x_{it}. \tag{3}$$

If the product is promoted in any period $< t$ then for any state $s_t \in S$ in period t, action set consist of only price promotion option for periods $\geq t$. If the product is not promoted in period $< t$ then the action set includes both the options for the periods $\geq t$.

The reward in each period $r(\cdot, \cdot)$ takes into consideration the selling price of each item R_a, linear ordering cost of c per unit ordered, holding cost of h per unit per unit time, shortage cost of s per unit short, and deterioration cost of b per unit.

Let,

$G(a, y_t) = (R_a[\mu_{ta}, i_t]^-) - (s[0, \mu_{ta} - i_t]^+) - [y_t \cdot c]$

where, $[a, b]^+$ is maximum (a, b) and $[a, b]^-$ is minimum (a, b).

Expected reward for any action (p, y_t) at any stage is given as,

Expected Reward = (Revenue - holding cost - shortage cost - ordering cost - deterioration cost) - fixed cost of promotion for promoted states.

$$r(s_t, (p, y_t)) = \sum_{\mu_{tp}=0}^{D} \Big[pr(\mu_{tp}) \Big\{G(p, y_t) - (h[0, i_t - \mu_{\mu_{tp}}]^+) - (b[0, x_{1t} - \mu_{tp}]^+)\Big\}\Big] - K \tag{4}$$

Similarly, for a state (n, y_t) only the fixed cost of promotion will be eliminated.

$$r(s_t, (n, y_t)) = \sum_{\mu_{tn}=0}^{D} \Big[pr(\mu_{tn}) \Big\{G(n, y_t) - (h[0, i_t - \mu_{tn}]^+) - (b[0, x_{1t} - \mu_{tn}]^+)\Big\}\Big] \tag{5}$$

We have assumed that for the last period there will not be inventory holding cost. Then the expected reward is given by,

$$r(s_N, (p, y_N)) = \left[\sum_{\mu_{Np}=0}^{D} [pr(\mu_{Np}) \{G(p, y_N) - (b[0, i_N - \mu_{Np}]^+)\}]\right] - K \tag{6}$$

Similarly, for a state (n, y_N) only the fixed cost of promotion will be eliminated.

$$r(s_N, (n, y_N)) = \sum_{\mu_{Nn}=0}^{D} \Big[pr(\mu_{Nn}) \Big\{G(n, y_N) - (b[0, i_N - \mu_{Nn}]^+)\Big\}\Big] \tag{7}$$

For $t \geq N + 1$, the reward $r(s_{N+1}) = 0 \quad \forall \ s_{N+1} \in S$

The total expected reward for a state s_t in a period t till the end of horizon is denoted by $Q_t(s_t,(a,y_t))$, where a is promotional related decision and y_t is the ordering quantity.

$$Q_t^*(s_t) = \max_{y_t \in Y_t, a} \left\{ r_t(s_t,(a,y_t)) + \left[\sum_{h \in S, \mu \in [0,D]} \left[\begin{array}{l} [pr(\mu_{ta}|(s_t,(a,y_t)))]* \\ [Q_{t+1}^*(h_{t+1})\ ] \end{array} \right] \right] \right\} \tag{8}$$

Where, $Y_t = \{0 \ldots I - j_t\}$, is set of actions for y_t, h_{t+1} is the resulting state in period '$t+1$' and $pr(\mu_{ta}|(s_t,(a,y_t)))$ indicates the probability of demand μ_{ta} when action (a,y_t) at state s_t is taken in period t. The optimal policy is found by recursive dynamic programming approach, starting with $t = N$ and solving the above set of equations, till t=0.

3.1 Numerical Analysis

Following are some examples to illustrate the results.

Example 1:- Let, $L = 5$, $I = 5$, $D = 5$, $c = 80$ $h = 1$, $s = 15$, $b = 40$, $K_p = 40$, $R_p = 96$, $R_n = 120$ and demand distributions as given in Table 1

	Demand distribution with higher variance							Demand distribution with small varia						
Demand Units	0	1	2	3	4	5	**CoV**	0	1	2	3	4	5	**CoV**
Promoted	0.0	0.25	0.25	0.25	0.25	0.0	**44.72**	0.0	0.0	0.5	0.5	0.0	0.0	**20**
Not Promoted	0.25	0.25	0.25	0.25	0.0	0.0	**74.53**	0.0	0.5	0.5	0.0	0.0	0.0	**33.33**

Table 1: Probability demand distribution

Inv. Vector Time period	1	2	3	4	5	6	7	8	9	10
With higher variance demand distribution										
(0, 0, 0, 0)	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-2	N-1
(1, 0, 0, 0)	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-0
(2, 0, 0, 0)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	P-2	P-1	P-0
(3, 0, 0, 0)	N-0	N-0	N-0	N-0	N-0	P-1	P-1	P-1	P-1	P-0
(4, 0, 0, 0)	N-0	N-0	N-0	P-0	P-0	P-0	P-0	P-0	P-0	P-0
(5, 0, 0, 0)	N-0	N-0	N-0	P-0	P-0	P-0	P-0	P-0	P-0	P-0
(1, 1, 0, 0)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	P-1	P-0
(2, 1, 0, 0)	N-0	N-0	N-0	N-0	N-0	N-0	P-1	P-1	P-0	P-0
(2, 0, 0, 1)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-1	P-0	P-0
With small variance demand distribution										
(0, 0, 0, 0)	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-1
(1, 0, 0, 0)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0
(2, 0, 0, 0)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-1	P-1	P-0
(3, 0, 0, 0)	N-0	N-0	N-0	N-0	P-0	P-0	P-0	P-0	P-0	P-0
(4, 0, 0, 0)	N-0	N-0	N-0	N-0	P-0	P-0	P-0	P-0	P-0	P-0

(5, 0, 0, 0)	N-0	N-0	N-0	N-0	P-0	P-0	P-0	P-0	P-0	P-0
(1, 1, 0, 0)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0
(2, 1, 0, 0)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	P-0	P-0
(2, 0, 0, 1)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	P-0	P-0

Table 2: Price promotion decision and ordering quantity with different demand distributions

Table 2 gives some of the representative inventory vectors with optimal ordering quantity and the promotion decision in the periods ranging from first (start of the horizon) to the tenth (end of the horizon). Here, first column under caption inventory vector represents the inventory vector for different total inventory with age-wise profile. Each element separated by comma represents the amount of inventory of a particular age *e.g.* the first element represents the number of units with one period of life remaining. Second element represents number of units with two periods of life remaining and so on. The summation of all quantities give the total inventory at the beginning of the period. Periods are represented rowwise starting from first period of the horizon to the last period of the horizon. In Table 2,

- P: Promotion and N : No promotion
- For example, N-2 represents, do not promote the price and order 2 more units.

As shown in the Table 2 beginning inventory vectors (4,0,0,0), (5,0,0,0) and (2,1,0,0) can be observed with higher number of periods being in price promoted states while vectors (0,0,0,0), (1,0,0,0) and (2,0,0,0) etc. shows that ordering quantity is also greater for the spreaded distribution. This is due to the fact that better knowledge of demand realization reduce the fraction of the goods that perishes.

Once a vector is observed in promoted state in period t then that vector is also observed in promoted state in the periods greater than t.

Inventory vector $x^{'} = \{x_1^{'},\ x_2^{'},\ \ldots,\ x_L^{'}\}$ is called ***adverse*** vector with respect to ***favorable*** vector $x = \{x_1,\ x_2,\ \ldots,\ x_L\}$ if the sum of the multiplication of the scalar weights (given to each vector element) and inventory quantity (corresponding to vector element) of vector $x^{'}$ is less than that of x. The weights given to the vector is proportional to the remaining life. Comparison is made between two inventory vectors having equal total inventory in the same period.

Example 2:- Let, $L = 5,\ \ I = 5,\ \ D = 5,\ \ c = 80,\ \ h = 1,\ \ s = 15,\ \ ,b = 40,\ \ K_p = 40,\ \ R_p = 96,\ \ R_n = 120$ and demand distribution as given in the Table 3.

Table 4 shows that expected reward is more for vector (0,0,2,0) than (0,2,0,0) and (0,0,2,0) while ordering quantity for vector (2,0,0,0) is more than (0,2,0,0) and (0,0,2,0) when promoted.

Period	(2, 0, 0, 0)			(0, 2, 0, 0)			(0, 0, 2, 0)		
	P/N	Q	y	P/N	Q	y	P/N	Q	y
1	N	368.995	1	N	458.1	1	N	473.164	1

2	N	326.853	1	N	415.917	1	N	430.946	1
3	N	284.715	1	N	373.743	1	N	388.763	1
4	N	242.59	1	N	330.853	1	N	346.757	1
5	N	198.54	1	N	289.876	1	N	304.75	1
6	P	167.15	2	N	251.098	1	N	262.39	0
7	P	155.2	2	N	220.04	0	N	220.04	0
8	P	154.65	0	N	154.65	0	N	154.65	0

Table 4: Results for different inventory vectors having the same total inventory

3.2 Observations

The following are the some observations made from the numerical results presented above.

- Age-wise distribution of inventory influences the ordering policy.
- For a given life of item and demand rate, as shortage cost increases the number of promotions for an inventory vector decreases and ordering quantity increases.
- Increase in holding cost and deterioration cost increases the number of promotions as supplier no longer willing to hold inventory whose life decreases with time, hence the ordering quantity also decreases.
- Narrow spread demand distributions give lesser number of promoted states as compared to the one with wide spread of distribution. Distribution can be called narrow spread if its coefficient of variation is less than that of other.
- As length of horizon increases the optimal policy of ordering and promotion becomes stable. It is only the periods towards the end of the horizon which give the complex decisions.
- By comparing the results from Table 4, we can say that, favorable vectors have more expected reward than the adverse vectors in the same period but, the ordering quantity for adverse vectors is greater than or equal to that of favorable vectors if promoted, otherwise it less than if both vectors are not promoted.

4 An Approximation Approach

As seen from above, the optimal policy is affected by different parameters, including the size of state space. The size of state space depends mainly on the life of item. For example, for a 3 period life of item, the state space will have 10 states, while for a 4 period life of item, it will have 56 states and for a 5 life period, it will have

Demand Units	0	1	2	3	4	5	**Mean**
Promoted	0.0	0.0	0.05	0.05	0.7	0.2	**4.05**
Not Promoted	0.05	0.05	0.7	0.2	0.0	0.0	**2.05**

Table 3: Demand distribution

126 states. Therefore, the size of the state space increases with life of items. For real life application, it is necessary to reduce the dimension of the state space. The approximations considered in literature are either of following types.

- Aggregation of fresher inventory [10]
- Aggregation of inventory of all ages, *i.e* considering total inventory [6]

In this context, we propose an approximation to reduce the size of state space by collapsing some of the fresher inventory elements of an inventory vector. The basis for approximation is our hypothesis that the expected outdating quantity is more sensitive to changes in older inventory than in fresher inventory. It can be possible to collapse some of the fersher inventory states to reduce the dimension of the state space. Therefore, an inventory vector $(x_1, x_2, \ldots, x_{L-2}, x_{L-1})$ may be collapsed into $(x_1, x_2, \ldots, x_{r-1}, \sum_{j=r}^{L-1} x_j)$, where, x_j is the number units having remaining life of j periods. This approximation is different from Nahmias' [10] approach, in which he suggests grouping of older inventory.

Let $f_t(\mathbf{x})$ be the expected number of units outdating, when on hand inventory vector is, $\mathbf{x} \equiv (x_1, x_2, \ldots, x_{L-1})$ and the current period is t. Thus,

$$\begin{aligned} f_t(\mathbf{x}) &= [x_{1t} - \mu_t]^+ \cdot pr(\mu_t \leq x_{1t}) + \ [x_{1t} + x_{2t} - (\mu_t + \ \mu_{t+1})]^+ \cdot pr(\mu_t + \mu_{t+1} \leq x_{1t} + x_{2t}) \\ &+ [x_{1t} + x_{2t} + x_{3t} - (\mu_t + \mu_{t+1} + \mu_{t+2})]^+ \cdot pr(\mu_t + \mu_{t+1} + \mu_{t+2} \leq x_{1t} + x_{2t} + x_{3t}) \\ &+ \ldots \ + \ [\sum_{i=1}^{L-1} x_i - \sum_{n=t}^{N} \mu_n]^+ \cdot pr(\sum_{n=t}^{N} \mu_n \leq \sum_{i=1}^{L-1} x_i), \end{aligned} \tag{9}$$

where, x_i is number of items having remaining life of i period and μ_t is demand in period t.

Then, as per our hypothesis,

$$\partial f_n(\mathbf{x})/\partial x_j > \partial f_n(\mathbf{x})/\partial x_i, \qquad i > j, \tag{10}$$

where, $\partial f_n(\mathbf{x})/\partial x_j$ is the change in expected outdating function with respect to inventory having remaining life of j periods.

That is, if older inventory element of an inventory vector is increased from x_k to $x_k + 1$, for some $1 \leq k \leq L - 1$, then the increase in expected number of outdating is more in this case than the case where fresher inventory is increased from x_m to $x_m + 1$, for some $k < m \leq L - 1$.

Let, $L - 1$ be the original dimension of an inventory vector and r be the dimension of a collapsed inventory vector. The maximum expected reward with actual shelf life of L periods is given by equation 8

Let $\hat{Q}^*_t(\hat{s}_t)$ be the reward for a collapsed inventory vector of dimension r. It is given by,

$$\hat{Q}^*_t(\hat{s}_t) = \max_{\hat{y}_t \in \hat{Y}_t, a} \left\{ \hat{r}_t(\hat{s}_t, (\hat{a}, \hat{y}_t)) + \left[\sum_{\hat{h} \in \hat{S}, \mu \in [0,D]} \left[\begin{array}{c} [pr(\mu_{ta} | (\hat{s}_t, (\hat{a}, \hat{y}_t)))] * \\ [\hat{Q}^*_{t+1}(\hat{h}(\hat{s}_t, \mu_t, \hat{y}_t)) \;] \end{array} \right] \right] \right\}. \quad (11)$$

Where, $\hat{s}_t$ is a collapsed inventory vector

$\hat{r}_t(.,.)$ is an expected one period reward from state $\hat{s}_t$.

$\hat{y}_t \in \hat{Y}_t = \{0 \dots I - j_t\}$ is an ordering quantity.

$\hat{s}_t = (x_1, \dots, x_{r-1}, \sum_{j=r}^{L-1} x_j)$

$\hat{h}$ is the transfer function given as,

$\hat{h}_j(y, \hat{s}, \mu_a) = (0, x_{j+1} - (\mu_a - \sum_{i=1}^{j} x_i)^+)^+ \quad i \le j \le r-2$

$\hat{h}_{r-1}(y, \hat{s}, \mu_a) = (0, y_{r-1} - (\mu_{r-1a} - \sum_{i=1}^{r} x_i)^+)^+$

and $\hat{S}$ is new state space for collapsed vectors. $\hat{s}_t, \hat{h} \in \hat{S}$. In this approximation the collapsing is done with the newer inventory. The items having life of more than r periods (L is the maximum life and $r < L$) are being treated as having life of r periods. Thus, it can be seen that the deterioration is being overestimated, hence, the approximated expected reward will be less than the actual one.

4.1 Numerical Analysis

An extensive numerical analysis is carried out to study the behavior of an optimal policy by varying various parameters, like demand pattern, cost parameters, *etc.* Different patterns of demand probability distributions considered are,

- Triangular distribution
- Two-sided Unimodal distribution
- One-sided Unimodal distribution
- Uniform Distribution
- General Non-unimodal Distribution

The approximation analysis is carried out with shelf life of more than 4 periods approximated to 4 periods. The life periods that are approximated are 5, 6, 7 and 8 periods of life over the different conditions of demand probability distribution and over the horizon of 10 periods and more. Following are some illustrative examples.

Example 4:- **Uniform Demand Distribution**

$I = 5, \quad D = 5, \quad c = 80 \quad h = 10, \quad s = 20, \quad b = 40, \quad K = 100, \quad R_p = 96, R_n = 120$ with the uniform demand distribution is given in Table 5.

The results are given in the Table 6.

Demand Units	**0**	**1**	**2**	**3**	**4**	**5**
Promoted	0.0	0.2	0.2	0.2	0.2	0.2
Not Promoted	0.2	0.2	0.2	0.2	0.2	0.0

Table 5: Demand Probability distribution

Inv. Vector \ Time period	1	2	3	4	5	6	7	8	9	10	Avg. %Error
(1, 0, 0, 1)	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-1	N-0	-
(1, 0, 0, 0, 1)	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-1	N-0	1.567
(1, 0, 0, 0, 0, 1)	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-1	N-0	1.877
(1, 0, 0, 0, 0, 0, 1)	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-1	N-0	1.933
(0, 0, 0, 3)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0	P-0	-
(0, 0, 0, 0, 3)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0	P-0	1.537
(0, 0, 0, 1, 2)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0	P-0	1.532
(0, 0, 0, 0, 0, 3)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0	P-0	1.860
(0, 0, 0, 0, 1, 2)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0	P-0	1.859
(0, 0, 0, 0, 0, 0, 3)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0	P-0	1.921
(0, 0, 0, 0, 0, 1, 2)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0	P-0	1.921

Table 6: Ordering quantity and promotional decision with uniform distribution

Average percentage error is an average taken over 10 periods of the percentage difference between the reward before collapsing inventory states and after collapsing inventory states.

Example 5:- **Triangular Demand Distribution**

$I = 5, \quad D = 5, \quad c = 60 \quad h = 10, \quad s = 15, \quad b = 50, \quad K = 50, \quad R_p = 80, R_n = 100.$
The demand distribution is uniform and given in Table 7.

Demand Units	**0**	**1**	**2**	**3**	**4**	**5**
Promoted	0.0	0.0	0.5	0.3333	0.1667	0.0
Not Promoted	0.0	0.5	0.3333	0.1667	0.0	0.0

Table 7: Demand Probability distribution

The results are presented in Table 8.

Inv. Vector / Time period	1	2	3	4	5	6	7	8	9	10	Avg. %Error
(1, 0, 0, 1)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0	N-0	-
(1, 0, 0, 0, 1)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0	N-0	0
(1, 0, 0, 0, 0, 1)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0	N-0	0
(1, 0, 0, 0, 0, 0, 1)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0	N-0	0
(0, 0, 0, 3)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	-
(0, 0, 0, 0, 3)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	0
(0, 0, 0, 1, 2)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	0
(0, 0, 0, 0, 0, 3)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	0
(0, 0, 0, 0, 1, 2)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	0
(0, 0, 0, 0, 0, 0, 3)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	0
(0, 0, 0, 0, 0, 1, 2)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	0

Table 8: Ordering quantity and promotional decision with triangular distribution

The error in optimal ordering quantity and promotional decision is 0. But, the error in the expected reward is positive, as approximation overestimates the deterioration.

Example 6: Approximation with shelf life of 2 and 3 period life.

Let, $I = 5$, $D = 5$, $c = 80$ $h = 1$, $s = 15$, $b = 40$, $K_p = 20$, $R_p = 96, R_n = 120$, with the demand distribution as given in Table 9 The solution for some of the

Table 9: Demand Probability distribution

Demand Units	**0**	**1**	**2**	**3**	**4**	**5**
Promoted	0.0	0.0	0.1	0.2	0.3	0.4
Not Promoted	0.1	0.2	0.3	0.4	0.0	0.0

representative vectors for the above parameters is given in Table 10

Inv. Vector / Time period	1	2	3	4	5	6	7	8	9	10	Avg. %Error
(2, 3)	N-0	N-0	N-0	N-0	N-0	P-0	P-0	P-0	P-0	P-0	-
(2, 3, 0)	N-0	N-0	N-0	N-0	N-0	N-0	P-0	P-0	P-0	P-0	4.231
(2, 1, 2)	N-0	N-0	N-0	N-0	N-0	N-0	P-0	P-0	P-0	P-0	10.449
(2, 0, 3)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	P-0	10.868
(2, 3, 0, 0)	N-0	N-0	N-0	N-0	N-0	N-0	P-0	P-0	P-0	P-0	4.723
(2, 0, 0, 3)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	P-0	12.352
(2, 0, 2, 1)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	P-0	12.094
(2, 1, 1, 1)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	P-0	11.612
(2, 3, 0, 0, 0)	N-0	N-0	N-0	N-0	N-0	N-0	P-0	P-0	P-0	P-0	4.774
(2, 0, 0, 0, 3)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	P-0	12.538
(2, 0, 0, 2, 1)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	P-0	12.511
(2, 0, 1, 1, 1)	N-0	N-0	N-0	N-0	N-0	N-0	N-0	N-0	P-0	P-0	12.473

Table 10: Approximation is poor with shelf life of 2 periods

4.2 Observations

Nandakumar and Morton [11] pointed out that as the life of the product increases the behavior of the ordering pattern comes close to that of non-perishable product. For approximation, using the above property it is necessary to find the suitable shelf life which not only gives lesser errors in reward value but also gives fairly close solutions for ordering and promotional decisions.

Some of the observations from the analysis are as follows:

1. The approximation gives poor results when the items having life of more than 3 periods are approximated to shelf life of 2 periods and 3 periods. With lesser life the risk of losing the stocks due to perishability and hence such approximations works well with narrow demand distribution.
2. The various experiments shows that the approximation to the life of 4 periods is good in various conditions of demand distributions and other parameters.
3. According to the definition of the adverse and favorable vector, if the approximated vector is good approximation for the favorable vector then it is also good approximation to adverse vector.
4. *Total average error*: The effectiveness of the algorithm compared by simulation over at different cost setting with different type of demand distribution distribution as discussed in the earlier section. The average percentage error of the approximation is in the range of 0-4%.
5. *Error variation with respect to life*: Error in approximation increases as the life of items increases. Since with increase in life increases the overestimation of deterioration. But the rate of increase in the error decreases as with life. Figure 1 shows the variation of error for shelf life of 5 periods to 8 periods

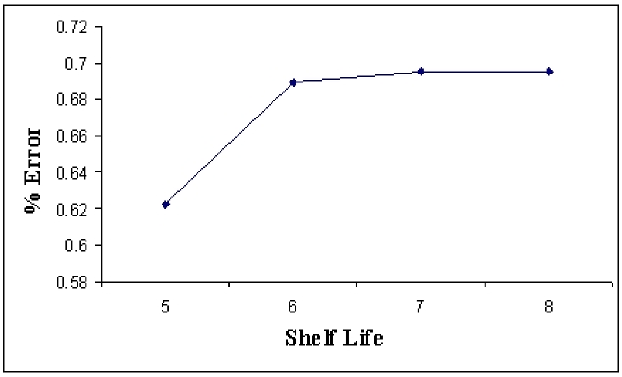

Figure 1: Percentage error in expected reward for different shelf lives

approximated to life of 4 periods.

6. *Error over the horizon*: It is observed that error decreases towards the end of the horizon. Also error is more with less beginning inventory.

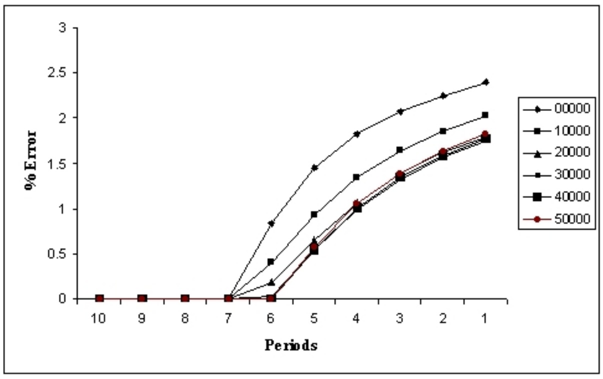

Figure 2: Percentage error in expected reward over horizon

7. *Error with favorable and adverse vector*: The pattern of difference in the error for the adverse and favorable vector is of mixed type.
8. *Error with different types of demand pattern*: Error for narrow spread distribution is less as compared to that of wide spread distribution.

4.3 Conditions Making Approximation poor

Demand Distribution

The distribution with narrow spreads generate better results than the one with larger coefficient of variation (coefficient of variation does not depend upon the whether the distribution is unimodal or not). This is mainly due to the fact that better knowledge about the demand realization reduces the fraction of the goods that perish. Approximation becomes poor with the non-unimodal distribution.

Example 7:- Let, $I = 5, \quad D = 5, \quad c = 80 \quad h = 1, \quad s = 15, \quad b = 70, \quad K_p = 0, \quad R_p = 96, R_n = 120$. Demand distribution is as given in Table 11. With such distribution

Demand Units	**0**	**1**	**2**	**3**	**4**	**5**
Promoted	0.05	0.05	0.4	0.05	0.4	0.05
Not Promoted	0.05	0.4	0.05	0.4	0.05	0.05

Table 11: Demand Probability distribution

the collapsed vector orders lesser quantity than the actual inventory vector.

Inv. Vector / Time period	1	2	3	4	5	6	7	8	9	10
(1, 0, 0, 1)	N-2	N-2	N-2	N-2	N-2	N-2	N-3	N-2	N-1	N-0
(1, 0, 0, 0, 1)	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-2	N-1	N-0
(1, 0, 0, 0, 0, 1)	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-2	N-1	N-0
(1, 0, 0, 0, 0, 0, 1)	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-2	N-1	N-0
(1, 0, 0, 2)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0	P-0
(1, 0, 0, 0, 2)	N-1	N-1	N-1	N-1	N-1	N-1	N-2	N-1	N-0	P-0
(1, 0, 0, 1, 0, 1)	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-1	N-0	P-0
(1, 0, 0, 0, 0, 0, 2)	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-1	N-0	P-0
(0, 0, 0, 1)	N-3	N-3	N-3	N-3	N-3	N-3	N-4	N-3	N-2	N-0
(0, 0, 0, 0, 1)	N-4	N-4	N-4	N-4	N-4	N-4	N-4	N-3	N-2	N-0
(0, 0, 0, 0, 0, 1)	N-4	N-4	N-4	N-4	N-4	N-4	N-4	N-3	N-2	N-0
(0, 0, 0, 0, 0, 0, 1)	N-4	N-4	N-4	N-4	N-4	N-4	N-4	N-3	N-2	N-0

Table 12: Results with non-unimodal distributions

Cost Parameters

The cost parameter affecting the approximation is mainly the holding cost. Example:
Let, $I = 5, \quad D = 5, \quad c = 80 \quad , s = 15, \quad b = 40, \quad K_p = 80, \quad R_p = 96, R_n = 120 \quad N = 10$ with the demand distribution as given in Table 13. The results with

Demand Units	**0**	**1**	**2**	**3**	**4**	**5**
Promoted	0.05	0.05	0.3	0.4	0.1	0.1
Not Promoted	0.1	0.1	0.5	0.1	0.1	0.1

Table 13: Demand Probability distribution

the above input parameters are given in the Table 14. The result from the Table 14 shows that the ordering quantity is less for the approximated vector than the actual vector.

It is observed that for certain range of the value of the holding cost the value of the ordering quantity is less that the actual vector. For the example with above input data the value of the holding cost from 4 to 6 gives poor results with respect to approximation.

Inv. Vector Time period	1	2	3	4	5	6	7	8	9	10
With the Holding Cost = 1										
(0, 1, 0, 0)	N-4	N-4	N-4	N-4	N-4	N-4	N-4	N-3	N-2	N-1
(1, 0, 0, 1)	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-1	N-0
(1, 2, 0, 0)	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-0	N-0
(0, 1, 0, 0, 0)	N-4	N-4	N-4	N-4	N-4	N-4	N-4	N-3	N-2	N-1
(1, 0, 0, 0, 1)	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-1	N-0
(1, 2, 0, 0, 0)	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-0	N-0
(0, 1, 0, 0, 0, 0)	N-4	N-4	N-4	N-4	N-4	N-4	N-4	N-3	N-2	N-1
(1, 0, 0, 0, 0, 1)	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-1	N-0
(1, 2, 0, 0, 0, 0)	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-0	N-0
With Holding Cost = 5										
(0, 1, 0, 0)	N-3	N-3	N-3	N-3	N-3	N-3	N-4	N-3	N-2	N-1
(1, 0, 0, 1)	N-2	N-2	N-2	N-2	N-2	N-2	N-3	N-2	N-1	N-0
(1, 2, 0, 0)	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-0	N-0
(0, 1, 0, 0, 0)	N-4	N-4	N-4	N-4	N-4	N-4	N-4	N-3	N-2	N-1
(1, 0, 0, 0, 1)	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-2	N-1	N-0
(1, 2, 0, 0, 0)	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-2	N-0	N-0
With holding cost = 7										
(0, 1, 0, 0)	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-2	N-1
(1, 0, 0, 1)	N-2	N-2	N-2	N-2	N-2	N-2	N-3	N-2	N-1	N-0
(1, 2, 0, 0)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0	N-0
(0, 1, 0, 0, 0)	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-2	N-1
(1, 0, 0, 0, 1)	N-2	N-2	N-2	N-2	N-2	N-2	N-3	N-2	N-1	N-0
(1, 2, 0, 0, 0)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0	N-0
(0, 1, 0, 0, 0, 0)	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-3	N-2	N-1
(1, 0, 0, 0, 0, 1)	N-2	N-2	N-2	N-2	N-2	N-2	N-3	N-2	N-1	N-0
(1, 2, 0, 0, 0, 0)	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-1	N-0	N-0

Table 14: Holding cost for which approximation is poor

5 Conclusion

The problem of finding optimal ordering policies for a fixed life item under the price promotion scenario is addressed. The problem is modeled using finite horizon dynamic programming approach. The approximation approach is quite useful for realistic applications of the finite horizon model to reduce the size of the state space. The computational analysis shows that shelf life of four periods gives good approximation to life of more than four periods in most of the settings. The average error of approximation lies between 0-4% for the reward value giving similar ordering quantity and promotional decision as the actual life problem. More complex scenarios can also considered, like multiple price promotions, *etc.*

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Appendix

Concavity of Reward Function

The reward function is formulated using dynamic programming in finite horizon case. Here the state variable is the beginning inventory vector s_t, where t being discrete time. Order quantity y_t, is control variable that is decision to be selected at each time epoch from given set. Demand μ_t is random variable, whose distribution is known. Concavity of the reward function is useful for determining the optimal value of control variable over countably infinite state space of random variable. The function will evaluate for optimality over the values of $y_t \in f(s_t)$ till the optimal reward value of $Q(s_t, (y_{t_i}, a))$ for y_{t_i} is greater than that of $Q(s_t, (y_{t_{i+1}}, a))$ for $y_{t_{i+1}}$, where, $y_{t_i} \in Y_t \;\; i = 1, 2, \ldots$ set of values y_t can take in period t. A stage can either be in promotion or no promotion. It is assumed that when t^{th} stage is promoted then $(t+1)^{th}$ stage is also promoted. Optimal expected reward function Q_t of period 't' is concave function in 'y_t' for a fixed beginning inventory vector s_t assuming that when present stage is promoted the subsequent stages are also promoted.

***Proof*:**

This is multi-period problem with backward dynamic programming formulation. Hence, the optimal expected reward for t^{th} period through N(horizon) is expected reward from t^{th} period plus optimal expected reward from period $t + 1$ through N (horizon). Decision variables of reward function are *"how much to order?"* i.e. order quantity and *"to promote or not to promote?"* i.e. promotional decision (assumes binary values). This gives three different cases under the assumption as cited below:

1. t^{th} stage is promoted and $(t-1)^{th}$ stage is also promoted.
2. t^{th} stage is not promoted and $(t-1)^{th}$ stage s also not promoted.
3. t^{th} stage is promoted and $(t-1)^{th}$ stage is not promoted.

Case I:

The t^{th} stage is promoted and $(t-1)^{th}$ stage is also promoted. Optimal expected reward is given as

$$Q_{t-1}(s_{t-1}) = \max_{y \geq 0} \left[g_{t-1}(\xi_{t-1}) + E_\mu \{ Q_t(\xi_t - \mu) \} \right] \tag{12}$$

Where,

$$g_t(\xi_t) = E_\mu \left\{ R_p[\mu, \xi_t]^- - s[\mu - \xi_t]^+ - h[\xi_t - \mu]^+ - b[\;\;]^+ - cy_t - K_p \right\} \tag{13}$$

Here, $\xi_t = s_t + y_t$, where

- ξ_t to order-up-to level for period t
- $s_t \;\; =$ initial inventory in period t
- $y_t \;\; =$ quantity ordered in period t

The terms $R_p[\mu, \xi_t]^- - cy_t$ is direct profit on quantity ordered which is concave function of y_t.

$s[\mu - \xi_t]^+ + h[\xi_t - \mu]^+$ addition of shortage and holding cost which charged linearly. Since, both are linear cost structure (in opposite direction) of y_t addition of both is a convex structure with respect to y_t as shown in Fig. This term is subtracted from profit i.e. the combination of these four terms

$$R_p[\mu, \xi_t]^- - cy_t - s[\mu - \xi_t]^+ - h[\xi_t - \mu]^+$$

is concave function.

Also, K_p is constant doesn't affect the concavity of the function, since it does not depend on the y_t.

Deterioration cost $b[\ ]^+$ is calculated differently in different periods as follows:
For $t = N($ Horizon$)$ $b[\ ]^+ = b[\xi_N - \mu]^+$ which is linear function in y_t
For $t < N($ Horizon$)$ $b[\ ]^+ = b[x_1 - \mu]^+$ which is independent of y_t hence constant.

Therefore, all terms in equation are concave combination hence the left hand side is also concave function for fixed s_t.
When $t = N$, $\quad Q_{N+1} = 0$ So, $Q_N(S_N)$ is concave as $g_N(\xi_N)$ is concave.
When $t = N - 1$,

$$Q_{N-1}(s_{N-1}) = \max_{y \geq 0} [g_{N-1}(\xi_{N-1}) + E_\mu\{Q_N(\xi_N - \mu)\}] \tag{14}$$

According to the Jensen' inequality, if f is concave function of X then

$$E[f(X)] \leq f(E[X]) \tag{15}$$

provided the expectation exists. From equation 14, $Q_t(.)$ is concave by induction hypothesis. To prove that, $E_\mu[Q_t(\xi - \mu)]$ is also concave that is to prove

$$\alpha E_\mu[Q_t(y_{t,1} + s_{t,1} - \mu_1)] + (1-\alpha)E[Q_t(y_{t,2} + s_{t,2} - \mu_2)]$$

$$\leq E_\mu[Q_t(\alpha y_{t,1} + (1-\alpha)y_{t,2} + \alpha s_{t,1} + (1-\alpha)s_{t,2} - \alpha\mu_1 + (1-\alpha)\mu_2)] \tag{16}$$

where, $0 < \alpha < 1$. Consider left hand side of the equation 16

$\alpha E_\mu[Q_t(y_{t,1} + s_{t,1} - \mu_1)] + (1-\alpha)E[Q_t(y_{t,2} + s_{t,2} - \mu_2)]$ can be written as

$E_\mu[\alpha Q_t(y_{t,1} + s_{t,1} - \mu_1) + (1-\alpha)Q_t(y_{t,2} + s_{t,2} - \mu_2)]$ which is

$$\leq E_\mu[Q_t(\alpha y_{t,1} + (1-\alpha)y_{t,2} + \alpha s_{t,1} + (1-\alpha)s_{t,2} - \alpha\mu_1 + (1-\alpha)\mu_2)] \tag{17}$$

since $Q_t(y_t + s_t - \mu)$ is concave. Hence, $E_\mu(\xi - \mu)$ is concave. Hence, inductively $Q_t(.)$ is concave function of y_t for fixed s_t.

Case II:

Here, t^{th} stage is not promoted and $(t-1)^{th}$ is also not promoted. This case is similar to the earlier case except that $K_p = 0$. Hence, optimal expected reward function is concave in y_t.

Case III:

Here, t^{th} stage is promoted and $(t-1)^{th}$ is not promoted.

$$Q_{t-1}(s_{t-1}) = \max_{y \geq 0} [g_{t-1}(\xi_{t-1}) + E_{\mu}\{Q_t(\xi_t - \mu)\}] \tag{18}$$

In the above equation $g_{t-1}()$ is reward when not promoted at $(t-1)^{th}$ stage which is itself a concave function form case II. Also, from case I Second term of right hand side of the equation is concave. Hence, optimal expected reward function is concave in y_t.

Corollary: Above optimality is defined with respect to ordering quantity y_t and for fixed inventory vector s_t. Since, when setup cost $K = 0$ and $Q_t()$ is concave there exist an optimal policy of the form

$$y_t^*(s_t) = \begin{cases} S_t - s_t & \text{if} \quad s_t < S_t \\ 0 & \text{if} \quad s_t \geq S_t \end{cases} \tag{19}$$

For a concave reward function of ordering quantity function the optimal policy is base-stock policy.