How To Find Your True Love? - From a Different Perspective

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How Can I Find True Love?

Try Google, and come up with:

- Learn from religious advice (love is what God says it is)
- Find your true love's zodiac sign through a five question quiz
- Consider phone counseling
  - (only to realize *you* are the real problem)

For all you romantics, there is Real Hope

Define:

True Love = The best person willing to date you

(Not always true, but still… convenience matters)

Ground Rules

- You only date one person at a time.
- A relationship either ends with you "rejecting" or "selecting" the other person.
- If you "reject" someone, the person is gone forever.
  - Sorry, old flames cannot be rekindled.
- You plan on dating some fixed number of people (N) during your lifetime.
- As you date people, you can only tell *relative* rank and not true rank.

Modeling of the game

- –*If you pick someone too early*, you are making a decision without checking out your options.
- –*If you wait too long*, you leave yourself with only a few candidates to pick from.
- The game boils down to selecting an optimal stopping time between playing the field and holding out too long.

What does the math say?

The Basic Advice

Reject a certain number of people, no matter how good they are, and then pick the next person better than all the previous ones.

How Many People Should You Reject?

Example Problem

Three Choices

A naïve approach is to select the first one

What are the odds?

1/3

Can We Make It Better?

YES!!!

Strategy:

Always reject the first person, and choose the next best one

RESULT

- 1 2 **3** Lose
- 1 3 **2** Lose
- 2 **1** 3 **Win**
- 2 3 **1** **Win**
- 3 **1** 2 **Win**
- 3 **2** 1 Lose

WOW!! A whopping 50%!!!

The Best Strategy

Reject the first 37 percent of the people you want to date and then pick the next person better anyone before

And you get your true love for 37% of time!!

Statistics

Number o people you want to date (N)	Number of people you should reject (k)
4	1
5	2
10	3
25	9
50	18
100	37

Mathematics Behind

- Find the value of k, in population N
- Model a function
  - P(k) = the probability of the best choice after rejecting k persons
- Find the optimal point

MATHEMATICS BEHIND

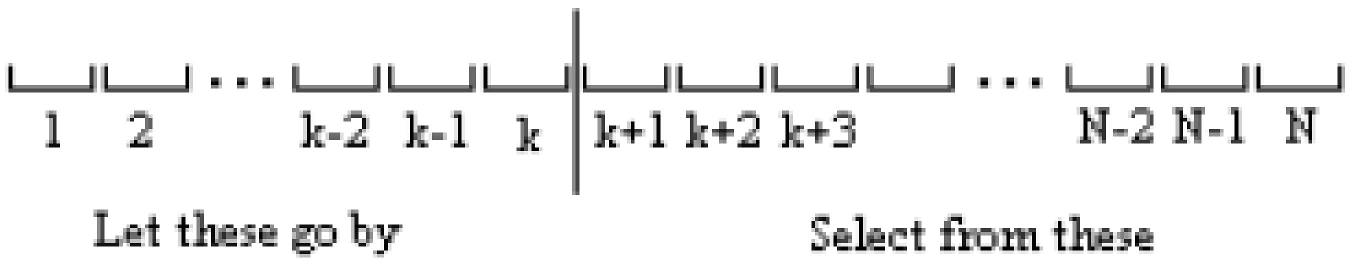

Figure 1: The Positions of the Candidates

Mathematics Behind

- P(k) = ∑(probability of the best in position n)(probability of being selected given in position n) [n=1 to N]

MATHEMATICS BEHIND – CALCULATING THE PROBABILITIES

- Let the best be in 1st place
- Probability of the best in 1 = 1/N
- Probability of selecting that = 0
- Total Probability for all upto "k" = 0

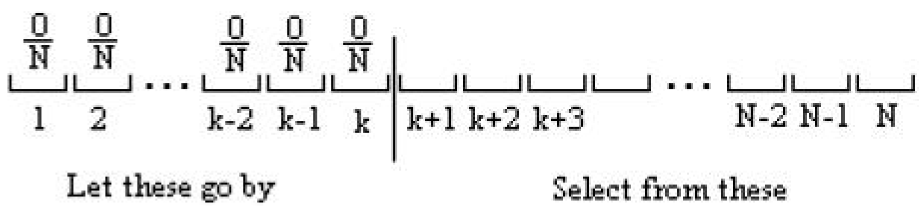

Figure 2: Probability of Success if Best Candidate is in Positions 1 to *k*

Mathematics Behind – Calculating the Probabilities

- Let the best is in k+1
- Probability of being the best = 1/N
- Probability of selecting it (given it is the best) = 1

Mathematics Behind – Calculating the Probabilities

Now a bit tougher problem

- How to find the probability if it is in place k+2?

- Probability of it being the best = 1/N
- Probability of selecting it (given it is the best) = k/k+1

Mathematics Behind – Calculating the Probabilities

- Similarly: for k+3,
- the probability = (1/N) * (k/k+2)

Mathematics Behind – Calculating the Probabilities

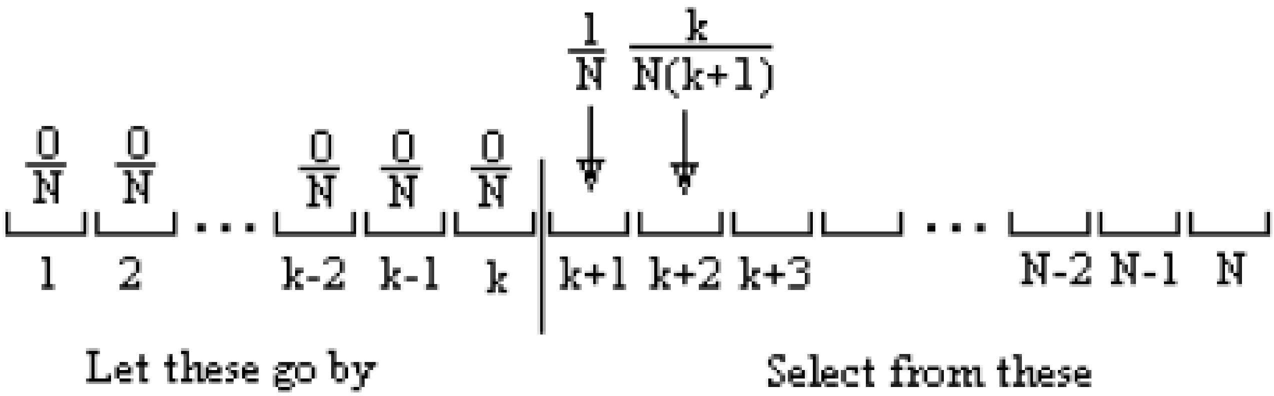

Figure 3: Probability for Positions $k+1$ and $k+2$

Mathematics Behind – Calculating the Probabilities

- So, we have

$$P(k)=\left(\frac{1}{N}\right)\cdot 0+\cdots+\left(\frac{1}{N}\right)\cdot 0+\left(\frac{1}{N}\right)\cdot 1+\left(\frac{1}{N}\right)\cdot\left(\frac{k}{k+1}\right)+\left(\frac{1}{N}\right)\cdot\left(\frac{k}{k+2}\right)+\cdots+\left(\frac{1}{N}\right)\cdot\left(\frac{k}{N-1}\right)$$

- Simplified

$$P(k)=\left(\frac{1}{N}\right)\left(1+\frac{k}{k+1}+\frac{k}{k+2}+\frac{k}{k+3}+\cdots+\frac{k}{N-1}\right),\ \text{for } k=0,1,2\ldots N-1$$

or

$$P(k)=\left(\frac{k}{N}\right)\left(\frac{1}{k}+\frac{1}{k+1}+\frac{1}{k+2}+\frac{1}{k+3}+\cdots+\frac{1}{N-1}\right),\ \text{for } k=1,2,\cdots N-1.$$

Mathematics Behind – Calculating the Probabilities

- Suppose N=5, take k=1,2,3,4

$$P(1)=\left(\frac{1}{5}\right)\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)=0.4167$$

$$P(2)=\left(\frac{2}{5}\right)\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)=0.4333$$

$$P(3)=\left(\frac{3}{5}\right)\left(\frac{1}{3}+\frac{1}{4}\right)=0.3500$$

$$P(4)=\left(\frac{4}{5}\right)\left(\frac{1}{4}\right)=0.2000$$

Mathematics Behind – Calculating the Probabilities

k	$N = 3$	$N = 4$	$N = 5$	$N = 6$	$N = 7$	$N = 8$	$N = 9$	$N = 10$	$N = 11$
0	0.333	0.25	0.200	0.167	0.143	0.125	0.111	0.100	0.091
1	$\boxed{0.500}$	$\boxed{0.458}$	0.417	0.381	0.350	0.324	0.302	0.283	0.266
2	0.333	0.457	$\boxed{0.433}$	$\boxed{0.428}$	$\boxed{0.414}$	0.398	0.382	0.366	0.351
3		0.25	0.350	0.392	0.407	$\boxed{0.410}$	$\boxed{0.406}$	$\boxed{0.399}$	0.390
4			0.200	0.300	0.352	0.380	0.393	0.398	$\boxed{0.398}$
5				0.167	0.262	0.318	0.353	0.373	0.384
6					0.143	0.232	0.290	0.327	0.352
7						0.125	0.208	0.265	0.305
8							0.111	0.189	0.244
9								0.100	0.173
10									0.091
k/N	0.333	0.500	0.400	0.333	0.286	0.375	0.333	0.300	0.364

Table1 : Best Positions for $N = 3$ to 11

Mathematics Behind

- How to determine the optimal value of k?
- Calculus?
  - Discrete Function

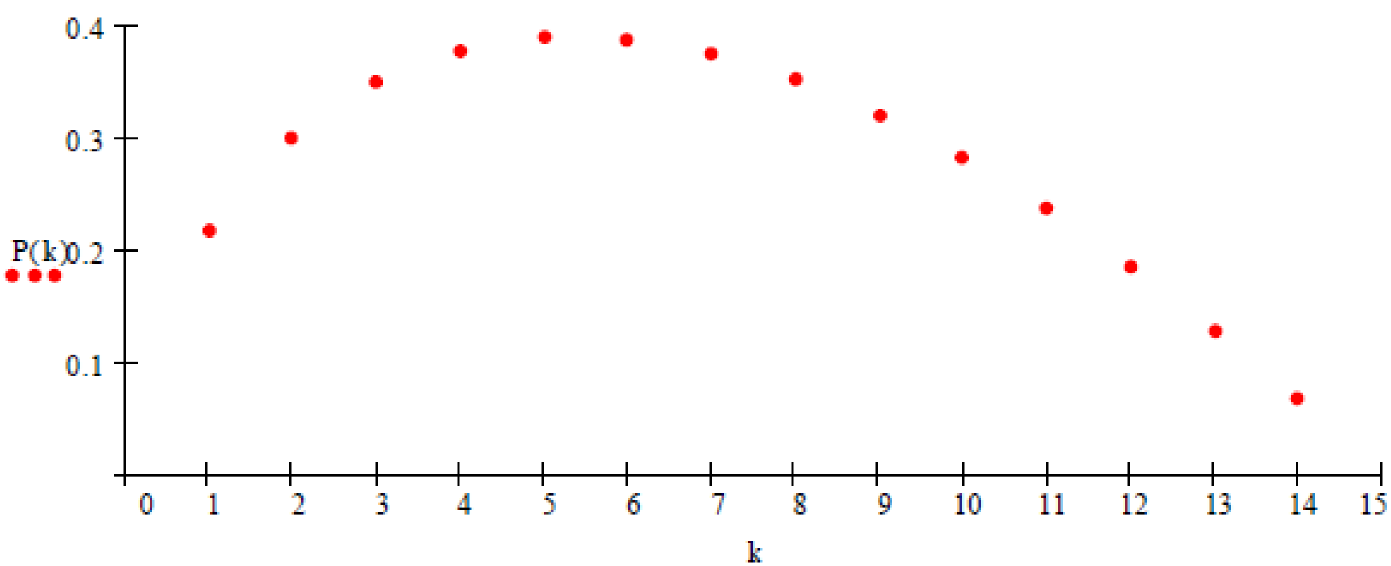

Figure 4: Plot of $P(k)$ against k for $N = 15$

Mathematics Behind

- How can we tell we are at the highest point on the graph?
- The value of k we want is first value of k for which the next probability is smaller than the preceding probability
- For which $P(k+1) - P(k) < 0$.
  - This process is similar to using the first derivative test in calculus.

Mathematics Behind

$$P(k+1)-P(k)=\left(\frac{k+1}{N}\right)\left(\frac{1}{k+1}+\frac{1}{k+2}+\cdots+\frac{1}{N-1}\right)-\left(\frac{k}{N}\right)\left(\frac{1}{k}+\frac{1}{k+2}+\cdots+\frac{1}{N-1}\right)$$

$$=\left(\frac{k+1}{N}\right)\left(\frac{1}{k+1}+\frac{1}{k+2}+\cdots+\frac{1}{N-1}\right)-\left(\frac{1}{N}\right)-\left(\frac{k}{N}\right)\left(\frac{1}{k+1}+\frac{1}{k+2}+\cdots+\frac{1}{N-1}\right)$$

$$=\left(\frac{k+1}{N}-\frac{k}{N}\right)\left(\frac{1}{k+1}+\frac{1}{k+2}+\cdots+\frac{1}{N-1}\right)-\left(\frac{1}{N}\right)$$

$$=\left(\frac{1}{N}\right)\left(\frac{1}{k+1}+\frac{1}{k+2}+\cdots+\frac{1}{N-1}-1\right).$$

So, if $P(k+1)-P(k)<0$, then $\left(\frac{1}{k+1}+\frac{1}{k+2}+\cdots+\frac{1}{N-1}-1\right)<0$.

Mathematics Behind

- A program can be easily written which terminates when $S_k = \left[\sum_{n=k+1}^{N-1} \frac{1}{n} \right] - 1$ will be negative

<u>Still, can't we find a definite "k" for all?</u>

MATHEMATICS BEHIND

- Consider the probability function

$$P(k)=\left(\frac{k}{N}\right)\left(\frac{1}{k}+\frac{1}{k+1}+\frac{1}{k+2}+\frac{1}{k+3}+\cdots+\frac{1}{N-1}\right).$$

- This approximates the area under the curve y=1/x, from x=k to x=N

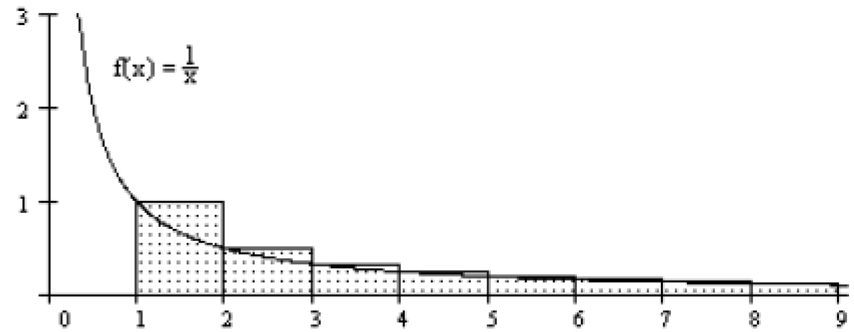

Figure 5: The area under $f(x)=\frac{1}{x}$

Mathematics Behind

- (1/k) + (1/k+1) +…+ (1/N-1) can be an approximation of
- ln(N)-ln(k) = ln(N/k)
- So, approximate function $P_1(k) = (k/N)\ln(N/k)$
- Poor approximation when N and k small
- Good approximation when large
- k/N=x

Mathematics Behind

- $P_1(x) = x\ln(1/x) = -x\ln(x)$
- $P'_1(x) = -1 - \ln(x)$
- If $P'_1(x) = 0 \Rightarrow x=1/e$

What is the probability of success?

$$P_1(e^{-1}) = -e^{-1}\ln(e^{-1}) =$$

$$\mathbf{1/e}$$

$$\mathbf{=0.368}$$

A Lesson Learned?

- Don’t settle too early!
- It’s not advisable to bank on your high school sweet-heart!
- Sounds odd, but it’s just too important to test the market and find that special person.
- Besides, this strategy improves a person’s odds from a pure random chance (10-20 percent) to almost 37 percent

Doesn't the Other Person Play the Game Too?

- The theory suggests you should not feel hurt if someone rejects you like this
  - You are likely an early victim
- To take advantage of the theory, you should consider whether the other person is ready to get serious
  - I guess this is why there are certain age clusters when people get married.

Loopholes

- You some times can rekindle an old flame
- You cannot date simultaneously, but you can often get to know many people at the same time
- You might be able to figure out a true love without having to date many more people
  - It happens

References

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Thank You For Your Patience!!