

Seminar on Ancient Egyptian Mathematics

Presented by

Ketki Kulkarni (IEOR)




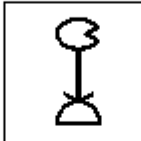



20th October 2009



Overview

- Inspiration
- Number System
- Arithmetic Operations
 - Addition and Subtraction
 - Multiplication and Division
- Fractions
- Some Numericals
- Trivia
 - Golden Ratio at Giza
 - Egyptian Calendar

Number System^[1]

	=1		=10		=100
			=1,000		
	=10,000		=100,000		=1,000,000

 = ?

Addition

$$\begin{array}{r} 27 \\ + 38 \\ \hline 65 \end{array}$$

Subtraction

$$\begin{array}{r} 63 \\ - 33 \\ \hline = 30 \\ - 5 \\ \hline = 25 \end{array}$$

Multiplication^[2]

41	59
1	59
2	118
4	236
8	472
16	944
32	1888

64 > 41so no further “doubling” required

$$41 - 32 = 9$$

$$9 - 8 = 1$$

Therefore, $41 = 32 + 8 + 1$

$$41 = 1.2^0 + 0.2^1 + 0.2^2 + 1.2^3 + 0.2^4 + 1.2^5$$

Division^[2]

1	65
2	130
4	260
8	520
16	1040

1. $1495/65$
 $16+4+2+1=23$
 $1495/65=23$

2. $1500/65$

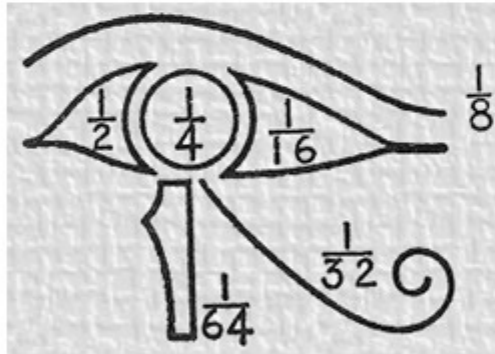
$$1500-65 < n \leq 1500$$

$$16+8+2+1=23$$

remainder=5
 $1500/65=23 \text{ } 1/13$

Fractions^[1]_[2]

- Only unit fractions
- $\frac{2}{3}$ being the exception
- Use of the 'Eye of Horace'
- All fractions denoted as sums of unit fractions



'Method of False Position'

- Take a first guess
$$x + x/4 = 15$$
- Multiply to get desired result


Some Numericals^[1]

Problem 50. A round field has diameter 9 khet.
What is its area?

Solution by Ahmes (Rhind Papyrus):

- Take away $1/9$ of the diameter, namely 1; the remainder is 8.
- Multiply 8 times 8; it makes 64.
- Therefore it contains 64 setat of land.
- The solution equivalent to taking

$$\pi = 4(8/9)^2 = 3.1605$$



Example 14. Example of calculating a truncated pyramid.
Base square side: 4 cubits, Top square side: 2 cubits,
Height: 6 cubits.

Area of the base: $4 \times 4 = 16$

- Area of the top $2 \times 2 = 4$.
- Product of the side of the base with the side of the top is computed: $4 \times 2 = 8$.
- The three added: $16 + 4 + 8 = 28$
- $1/3$ of the height: 2
- the product : 56
- $V = h (a^2 + ab + b^2)/3$

Trivia^[3]

Great Pyramid:

- The faces at $51^\circ 50' 35''$ with base. The secant 1.61806 remarkably close to the golden ratio 1.618034.
- Cotangent of the slope angle of $51^\circ 50' 35''$ is very close to $\pi/4$.
- A numerical coincidence: the square root of the golden ratio times π is close to 4, in fact this product is 3.996168.



Calendar

- The beginning of the year was chosen as the heliacal rising of Sirius, the brightest star in the sky.
- The Nile flooded shortly after this. The year computed to be 365 days long
- A more accurate value of 365 $\frac{1}{4}$ days worked out for the length of the year but the civil calendar never changed



References

1. <http://www.gap-system.org/~history/HistTopics/Egyptian>
2. http://math.suite101.com/article.cfm/the_mathematics_o
3. The Impressive Numerical History of an Ancient Civilization by Isaac M. McPhee