Sudoku

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Outline

- What is Sudoku?
- History
- Challenges
- Maths of Sudoku
- How to generate Sudoku?
- Solving Sudoku

What is Sudoku?

				3	6	1		
3	5			1	2		4	
6	4				8			
7	3				1	9		
4			3	6	9			8
		6	2				3	1
			6				1	7
	8		1	4			2	3
		3	8	2				

Source: Websudoku [7]

What is Sudoku?

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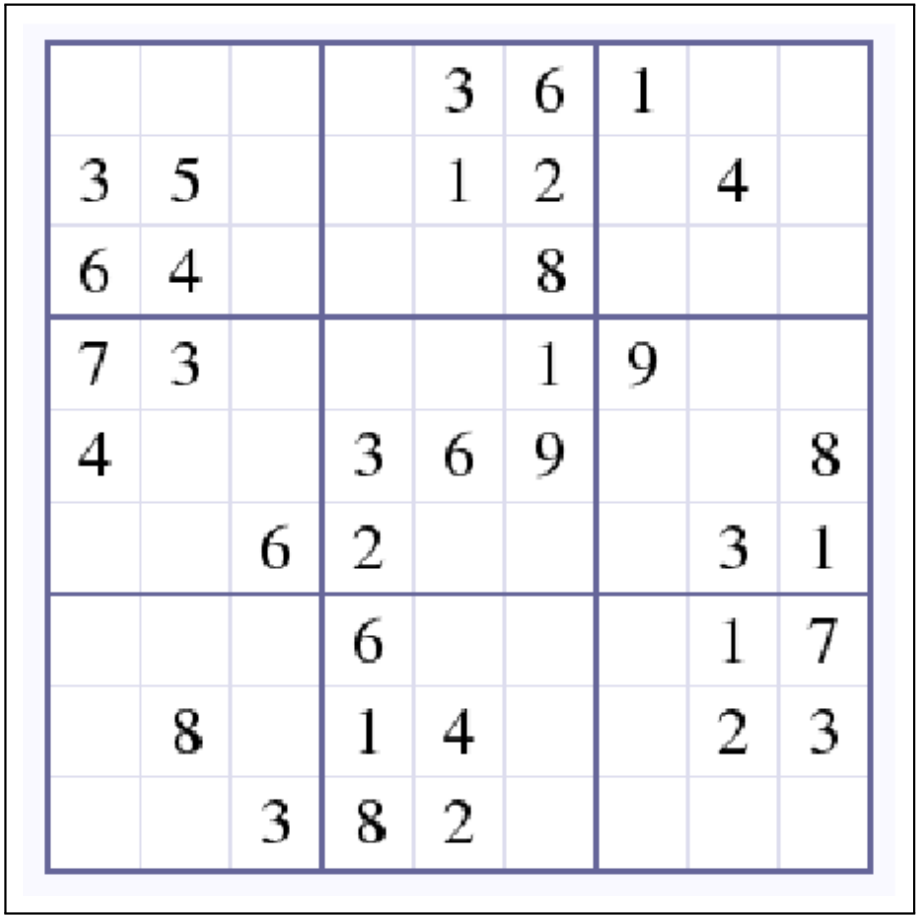

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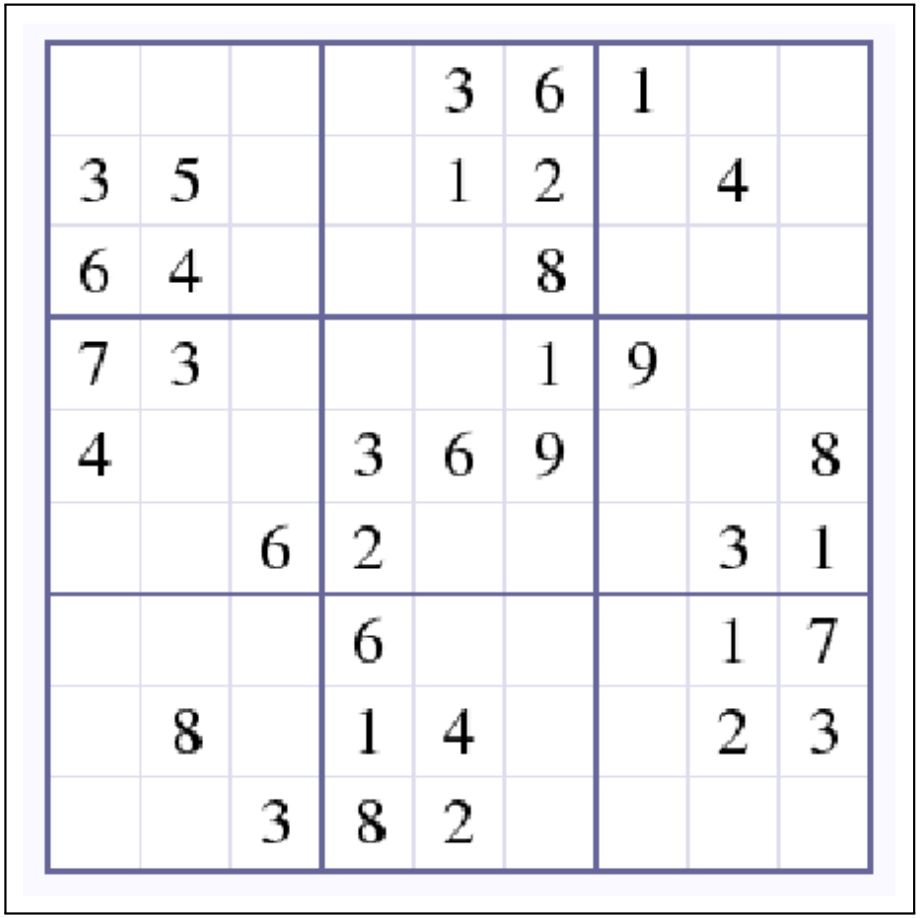

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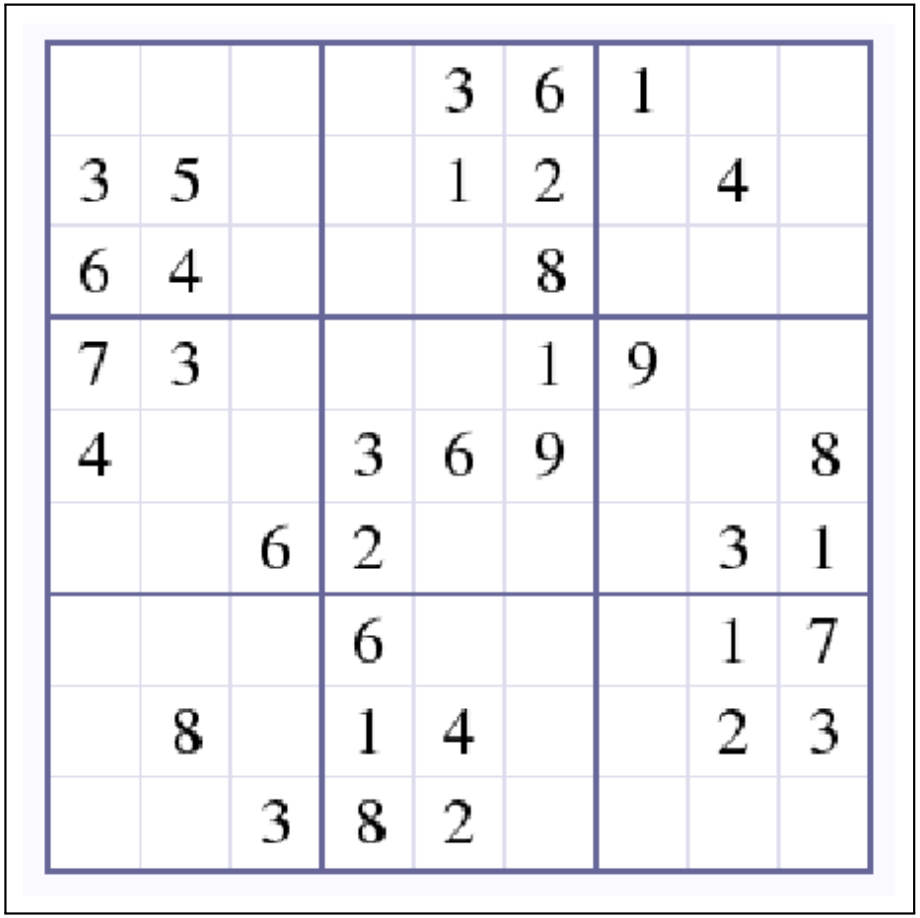
- Now this puzzle is very popular and available in internet, books, newspapers, mobiles etc.



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- By 2004, many interested people developed computer programs to generate them.
- *The Times* newspaper in London published the first game November 12, 2004. Within a few months, other British and US newspapers began publishing their own Sudoku puzzles.

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- Uniqueness cannot be guaranteed even if 77 out of 81 grids are known. The inverse problem - the fewest givens that render a solution unique - is unsolved.
- The maximum number of independent clues is 33 and the minimum is 17 (not yet proved).

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3. 9×9 grid can be studied as $Z_3 \oplus Z_3$.
4. Genereal Sudoku problem can be formulated as a binary integer linear program (BILP)[1].

Sudoku BILP formulation

Decision variable

$$x_{ijk} = \begin{cases} 1, & \text{if element } (i,j) \text{ of the } n \times n \text{ Sudoku matrix contains integer } k \\ 0, & \text{otherwise.} \end{cases}$$

$$\min \quad \mathbf{0}^T \mathbf{x}$$

$$s.t. \quad \sum_{i=1}^{n} x_{ijk} = 1, \quad j{=}1{:}n,\ k{=}1{:}n \quad \text{(only one } k \text{ in each column)}$$

$$\sum_{j=1}^{n} x_{ijk} = 1, \quad i{=}1{:}n,\ k{=}1{:}n \quad \text{(only one } k \text{ in each row)}$$

$$\sum_{j=mq-m+1}^{mq} \sum_{i=mp-m+1}^{mp} x_{ijk} = 1, \quad k{=}1{:}n,\ p{=}1{:}m,\ q{=}1{:}m$$

(only one k in each submatrix)

$$\sum_{k=1}^{n} x_{ijk} = 1 \quad i{=}1{:}n,\ j{=}1{:}n \quad \text{(every position in matrix must be filled)}$$

$$x_{ijk} = 1 \quad \forall (i,j,k) \in G \quad \text{(given elements } G \text{ in matrix are set "on")}$$

$$x_{ijk} \in \{0,1\}$$

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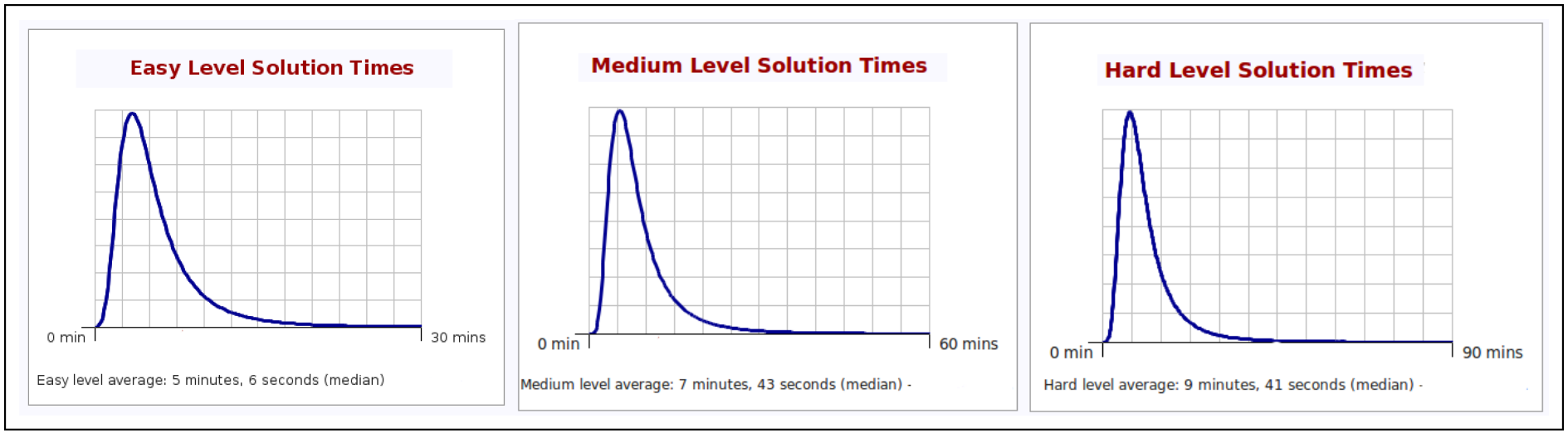
2. Creating New Puzzles from Old Puzzles.
   - Relabeling symbols
   - Row, stack, column permutations
   - Reflection, transposition or (1/4 turn) rotation (2)

Classification

- In general Sudoku puzzles are classified as Easy, Medium and Hard based on number of given clues.
- The difficulty of a puzzle is related to the depth of thinking required.
- In my experience, easy puzzle can be solved systematically, medium puzzles required a guess and hard puzzles need two guesses.



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- Naked and Hidden Pairs, Triplets, Quads,$\cdots$.

Shall we solve?

5			7		9			2
9	6			1			7	5
		3				6		
	8		4		6		1	
6								4
	9		2		8		5	
		9				2		
1	5			8			4	7
3			6		1			9

Sudoku addiction

- Sudoku, which literally means single, celibate, unmarried the precise description of people who become hopelessly addicted.
- In June 2008 an Australian drugs-related jury trial costing over AU$1,000,000 was aborted when it was discovered that five of the twelve jurors had been playing Sudoku instead of listening to evidence.
- Some unofficial reports claim that addiction to Sudoku caused Apollo 13 and Titanic disaster.

Thank You

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