A heuristic to generate input sequence for simulated annealing to solve VRP

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Abstract: This paper addresses the issue of input sequence, which affects the results of simulated annealing (SA) for vehicle routing problem (VRP). Also, a heuristic has been proposed which improves the result by 1.7% than the well known and frequently used input sequence. With its ability to provide solutions of good quality at low computing times, the proposed heuristic has ample scope for application as an automated scheduler when implemented at the site of a logistics-planning cell.

Keywords: heuristic; shrink wrap algorithm; simulated annealing; SA.

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1 Introduction

Solving a vehicle routing problem (VRP) consists of finding the shortest (or a nearly shortest) path connecting a number of locations, such as cities to be visited by a vehicle in its despatch route. VRP is very similar to the notorious travelling salesman problem (TSP) which is typical of a large class of 'hard' optimisation problems that have intrigued mathematicians and computer scientists for years. Hence, it is important to determine the proper and best method to solve the VRP in a reasonable computational time.

In this paper, SA is considered for optimising the VRP. It is well known fact that, even though SA goes beyond the local optimum, it results in the near optimal solution. Depending upon the input sequence of the VRP, the output of SA varies much. Here, we propose a heuristic to generate the input sequence for SA. The results of the proposed heuristic are found to be better than the shortest path input sequence.

2 Literature review

2.1 Basic VRP and algorithms

The single-depot multiple-vehicle node routing problem (sometimes called the vehicle routing problem – VRP) prescribes a set of delivery routes for vehicles housed at a central depot, which services all the nodes and minimises total distance travelled. The formulations of these problems were stated by Bodin et al. (1983). This problem can be extended by adding side-constraints such as time windows. Most solution strategies for the standard VRP can be classified as one of the following approaches:

- Cluster-first route-second procedures, which cluster demand nodes and/or arcs first, then design economical routes over each cluster as a second step. Example of this idea was given by Gillet and Miller (1974).
- Route – first cluster – second procedures, which work in a reverse sequence in comparison with cluster – first route – second. Golden et al. (1982) provided an algorithm that typifies this approach for a heterogeneous fleet size VRP.

- Savings or insertion procedures, which build a solution in such a way that at each step of the procedure, a current configuration that is possibly infeasible. The alternative configuration is one that yields the largest saving in terms of some criterion function, such as total cost, or that inserts least expensively a demand entity in the current configuration into the existing route or routes. Examples of these procedures can be found in Clarke and Wright (1964).
- Improvement or exchange procedures, such as the well-known branch exchange heuristic which always maintain feasibility and strive towards optimality. Other improvement procedures were described by Potvin and Rousseau (1995), including Or-opt exchange method in which one, two, three consecutive nodes in a route will be removed and inserted at another location within the same or another route; *k*-interchange heuristic in which *k* links in the current routes are exchanged for *k* new links; and 2-opt procedure which exchanges only two edges taken from two different routes.
- Mathematical programming approaches, which include algorithms that are directly based on a mathematical programming formulation of the underlying routing problem. An example of this procedure was given by Fisher and Jaikumar (1981). Christofides et al. (1981) discussed Lagrangean relaxation procedures for the routing of vehicle.
- Interactive optimisation, which is a general purpose approach in which a high degree of human interaction is incorporated into the problem solving process. Some adaptations of this approach to VRPs are presented by Krolark et al. (1970).
- Exact procedures for solving VRPs, which includes specialised branch and bound (B&B), dynamic programming and cutting plane algorithms. Kolen et al. (1987) described a B&B method for the VRP with time windows (VRPTW). Other applications of the above algorithms for VRPs were discussed in details by Christofides et al. (1981) and Laporte et al. (1992).
- Heuristic approaches: e.g., simulated annealing (SA) or tabu search (TS). For example, Bramel and Simchi-Levi (1995) introduced the location-based heuristic for general routing problem, which is based on formulating the routing problem as a location problem – commonly called the capacitated concentrator location problem. This location problem was subsequently solved and the solution was transformed back into a solution to the routing problem. The method incorporates many routing features into the model. Thangiah et al. (1996) presented a two-phase heuristic approach to solve the VRP with backhauls and time windows constraints. First, the initial solution was obtained by using Solomon's insertion heuristic procedure, and was improved through a combination of λ-interchanges and 2-opt exchanges to find a good solution for the problem Taillard et al. (1997) introduced a TS heuristic for the VRP with soft time windows. The original problem was converted into the VRP with hard time windows by adding large penalty values, and then an exchange procedure was used to swap sequences of consecutive customers between two routes. Finally, a selection procedure was used to find to best overall solution.

The following is the literature summary for the methodologies used to solve the mTSP and VRP.

Table 1 Literature summary

Variants	*Methodology*	*Author and year*
MTSP	Heuristic approach	Gorenstein (1970)
	Heuristic approach	Angel et al. (1972)
	Branch and bound algorithm	Svestka and Huckfeldt (1973)
	Linear admissible transformations	Berenguer (1979)
	Constraint relaxation approach	Laporte and Nobert (1980)
	Branch and bound algorithm	Radharamanan and Choi (1986)
	Branch and bound algorithm	Bezalel and Kizhanathan (1986)
	Heuristic approach	Okonjo-Adigwe (1988)
	Multi stage Monte Carlo optimisation	Conley (1990)
	Crossbar Hopfield neural network	Lo and Bavarian (1993)
	Tabu search	Golden et al. (1997)
	Adaptive neural network	Torki et al. (1997)
	Competition based neural network	Somhom et al. (1999)
	Modified genetic algorithm	Tang et al. (2000)
	Genetic algorithm	Carter and Ragsdale (2002)
mTSPMD	Integer linear programming	Kara and Bektas (2006)
mTSPTW	Lagrangean relaxation method	Jacques et al. (1988)
	Decomposition heuristic	Calvo and Cordone (2003)
	Genetic algorithm	Saleh and Chelouah (2004)
	Graph theory – precedence graphs	Mitrović-Minić and Krishnamurti (2006)
mTSPMP	Heuristic approach	Gilbert and Hofstra (1992)
mTSPFC	Transformation procedure	Saman and Manfred (1977)

2.2 TSP – perspective

The importance of the TSP does not arise from an overwhelming demand of salespeople to minimise their travel distance, but rather from a wealth of other applications, many of which seem to have nothing to do with the TSP at first glance. Real world application areas include: vehicle routing, circuit board drilling, VLSI design, robot control, X-ray crystallography, machine scheduling, and computational biology.

3 Connections between TSP and VRP

The TSP is the problem based on a salesman who, starting from a home base, must visit each of a specified set of cities and return home. The problem is to visit the cities in an order that minimises the total distance of the tour, i.e., the sequence of cities visited. More precisely, if we let the n-tuple (i_1, i_2, i_3, …, i_n) represent the tour that begins in city 0 and proceeds in order to cities i_1, i_2, i_3, …, i_n, then returns to city 0, and if $D(i, j)$ is the distance of travelling from city i to city j, we must find a permutation of i_1, i_2, i_3, …, i_n, that minimises the sum $D(l, i_2) + D(i_2, i_3) + \ldots + D(i_n - l, i_n) + D(i_n, l)$. VRP is very similar to this travelling salesperson problem (TSP), which consists of determining a route in which the vehicle will cover all the cities and turn back to a home city (depot). The role of VRP is significant in the supply chain management.

4 VR problem

The problem is to cover the cities in an order that minimises the total distance of the tour, i.e., the sequence of cities visited. We view the D(i, j) as a symmetric distance matrix D (i.e.,) D(i, j) = D(j, i). A solution to the VRP means a method of obtaining the optimal path in a reasonable amount of computer time. Ideally, we would like an algorithm that solves the problem in a short period of time.

There have been two classes of approaches to the TSP. The approaches in one class are exact in that they guarantee the optimal solution; whereas those in the other are approximate or heuristic in that they give solutions not guaranteed to be optimal but require fewer steps to compute.

4.1 Greedy heuristics

In principle, one can enumerate all possible tours, but, in practice, the number of tours is so staggeringly large (roughly N factorial) that this approach is useless. For large N, no one knows an efficient method that can find the shortest possible tour for any given set of points. Many methods have been studied that seem to work well in practice, even though they are not guaranteed to produce the best possible tour. Such methods are called heuristics. In this paper too, we propose a heuristic which generates an input sequence for SA which proves to work better.

5 SA for an optimisation problem

The attractiveness of using the SA approach for combinatorial optimisation problems is that transitions away from a local optimum are always possible when the temperature is non-zero. As pointed out by Kirkpatrick et al. (1983), the temperature is merely a control parameter so we no longer require Boltzmann's constant. In keeping with Kirkpatrick's et al. terminology, we will refer to this control parameter as a temperature. In discrete optimisation problems, it controls the probability of accepting a tour length increase. As such, it is expressed in the same units as the objective function. In implementing the approach, the 2-opt procedure is used for rearranging a tour.

5.1 Neighbourhood generation

The neighbourhood creation scheme is used in the model. For a given sequence σ, this neighbourhood is defined as the set of sequences obtained by moving any product from its actual position in $\sigma 1$ to any other position. The number of neighbourhood depends upon the application and the common formula used is $2(n - 1)$ numbers of neighbourhoods. From those neighbourhoods, the best one is selected according to the objective function.

5.2 Simulated algorithm for the process

Start with an initial sequence σ_0 and set $\sigma^* = \sigma_0$

Choose an initial temperature τ_0 and set $\tau = \tau_0$

Set temperature change counter $t = 0$

Repeat until $N < N_{\max}$

Set repetition counter $r = 0$

Repeat until $r = r_t^{\max}$

Generate $2(n - 1)$ neighbourhood sequences of the current sequence σ_0

  Choose the minimum among all neighbourhood sequence σ'

  Calculate $\delta = Z(\sigma_0) - Z(\sigma^1)$, the improvement in the objective function value if σ' is retained.

IF $\delta > 0$ OR Random (0, 1) $< e^{-\delta/t}$ THEN set $\sigma_0 = \sigma_1$

IF $z(\sigma^1) < z(\sigma^*)$ THEN set $\sigma^* = \sigma^1$ and $z(\sigma^*) = z(\sigma^1)$.

$R = r + 1$

End repeat

$T = t + 1$

$\tau = \alpha * \tau$

End repeat

5.3 Factors affecting implementation of SA for VRP

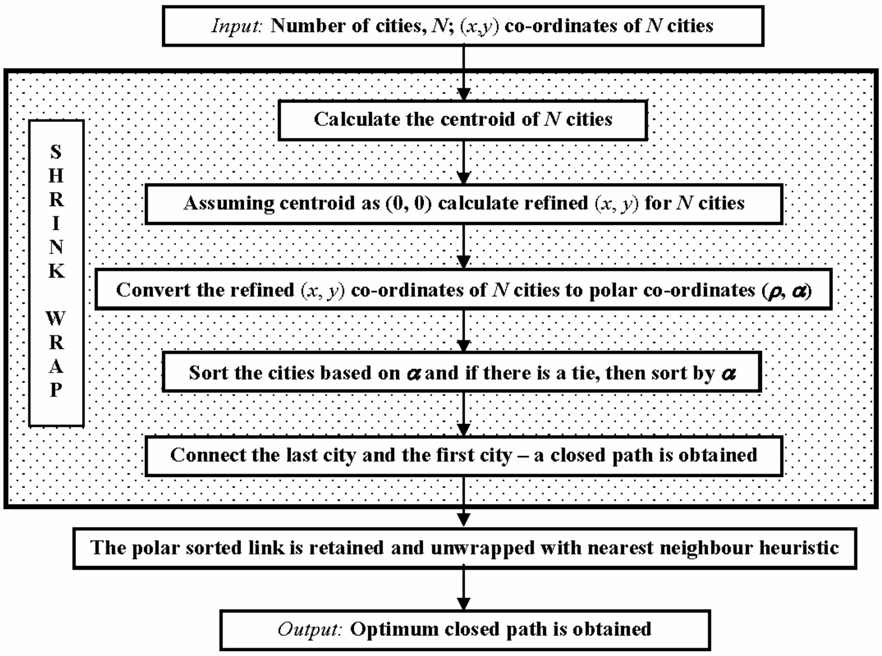
For proper implementation of the SA, we have to analyse the initial temperature, annealing schedule, cooling rate in general and the input sequence also for a VRP in particular. In order to get better results, a proper input sequence is required, since most of the time SA results in near optimal solution. Hence, in this paper, we focus on generating the input sequence for SA.

6 Proposed heuristic for solving VRP

The proposed SA for VRP is explained with a proper heuristically developed input sequence which uses shrink-wrap algorithm. Based on this input sequence, further sequences are generated randomly and the sequences are improved with the concept of SA to arrive at the better solutions in a reasonable computation time.

The classical concept of generating a sequence of solutions for a particular temperature T is used in the algorithm. A new solution is generated as a slight perturbation of the current state by using a single insertion neighbourhood creation scheme by creating $2(n - 1)$ neighbourhoods. If the difference in the total cost, δ between the current state and the slightly perturbed one is negative, then the process is continued from the new state. If $\delta >= 0$, then the perturbation of acceptance of the perturbed state is given by the Metropolis. The heuristic for the generation input sequence for SA is shown in Figure 1.

Figure 1 Proposed heuristic for generation of input sequence



7 Heuristic for generating input sequence

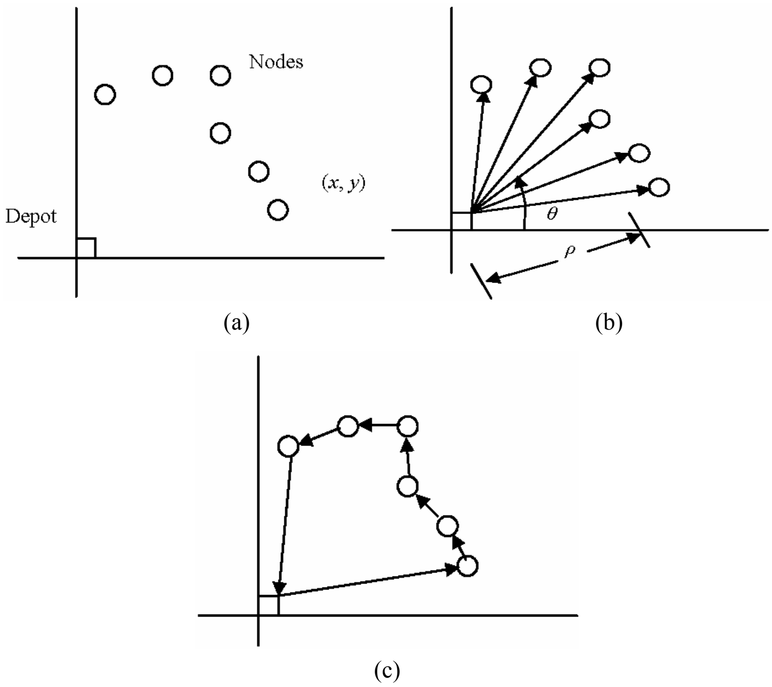
7.1 Shrink wrap algorithm

For the given cities, we orient the nodes along a path using the shrink-wrap algorithm. The nodes are mapped on polar coordinates, sorted by angle, then by distance (θ first, then ρ) and arranged in ascending order. This gives the path to be traversed within each cluster. Briefly, the steps are:

1. Convert the coordinates of all the nodes to polar coordinates (θ, ρ).
2. Sort the list of nodes in ascending order of θ. If the θ of two nodes are equal, then sort by ρ.
3. Join the nodes in the sorted list in the same sequence to obtain the tour for each cluster. The steps are shown in Figure 2.

The cities oriented using SA is again unwrapped by doing small perturbations between the cities.

Figure 2 Orientation of cities using shrink-wrap algorithm (a) map Cartesian coordinates as polar coordinates (b) sort by polar coordinates (c) link sorted list to determine tour



8 Problem formulation

8.1 Euclidean VRP

Euclidean VRP is the VRP with the distance being the ordinary Euclidean distance.

Euclidean VRP is a particular case of VRP in which D(i, j) = D(j, i). The distance between any two cities with coordinates (x_1, y_1) and (x_2, y_2) is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

We consider here 25 cities in the Euclidean map (Figure 3) and a single vehicle has to travel in an optimised path.

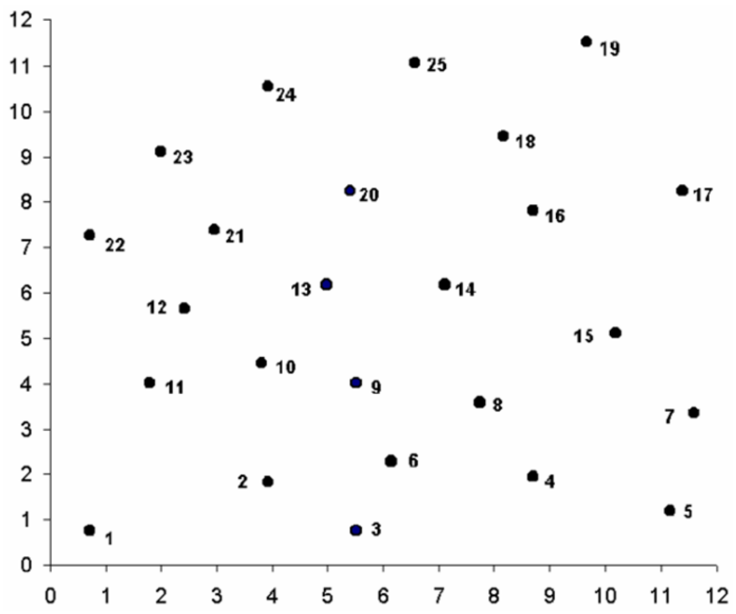
For solving the problem, the following assumptions are made.

1 no capacity constraints

2 no cost constraints

3 the vehicle has to visit a particular city exactly once.

Figure 3 Input data (see online version for colours)



9 Results and discussion

The results obtained using the various input sequences are as follows.

Table 2 Results of different heuristics

S. no.	*Mode of generating input sequence*	*Input route distance*	*Output route distance*
1	Random	80.1	70.6
2	Nearest neighbour	65.3	59.5
3	Shrink wrap algorithm	73.3	61.7
4	Proposed heuristic	61.3	58.5

The test data shows that the input sequence has a major effect on the application of SA for VRP. Also, here SA is capable of producing the results nearer to the given input value. So, as far as possible the input must be optimised.

If a random input sequence is given it may result in very local optimum and the results are erroneous as shown in Figure 4. Already this method is not used anywhere, but the frequently used methodology is as follows: Starting from the depot, the vehicle will move to the nearest city and again considering the second point as the reference it will move again to the nearest. The process repeats until the vehicle visits all the cities. This process seems to be better than the random selection and the output is shown in Figure 5. The shrink wrap algorithm is also given as input directly the result, Figure 6 shows an increase in distance compared with nearest neighbour input sequence generation. By doing a slight perturbation in the route generated by shrink wrap algorithm, we get a new route which gives better results, Figure 7 when compared with the other methods input sequences.

Figure 4 Output obtained for random input sequence (see online version for colours)

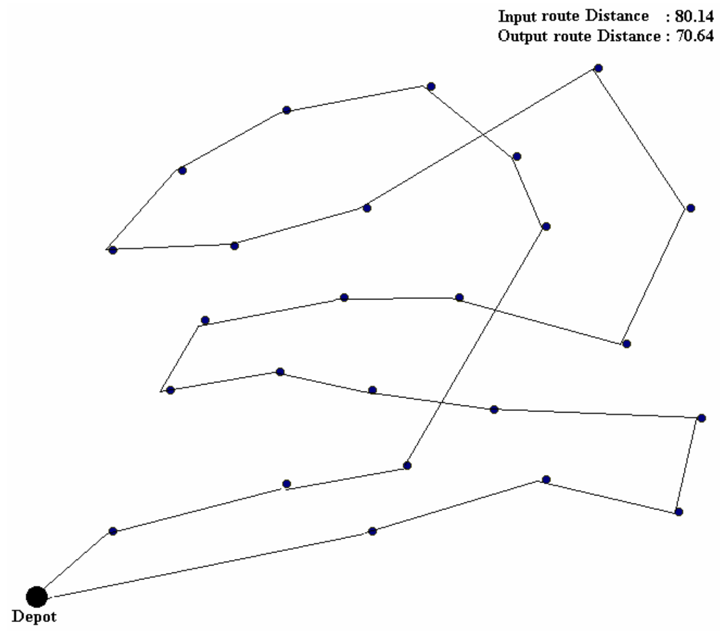


Figure 5 Output obtained for nearest neighbour input sequence (see online version for colours)

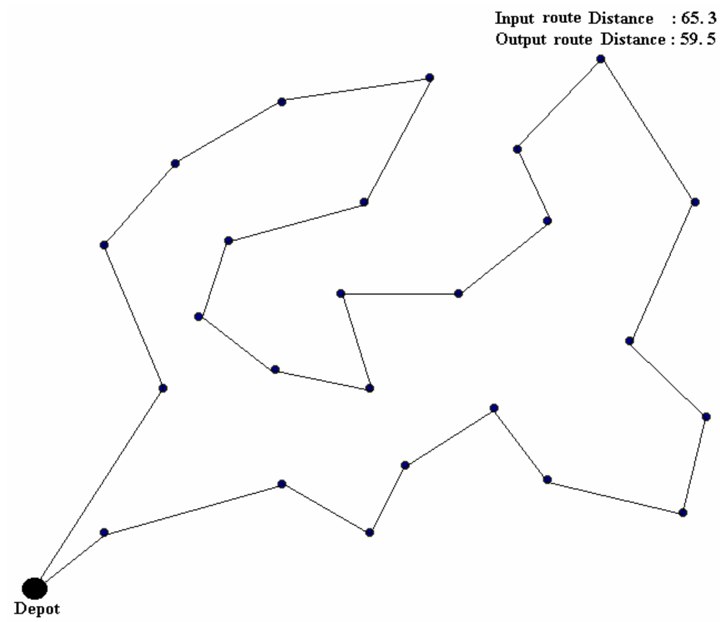


Figure 6 Output obtained for shrink wrap algorithm input sequence (see online version for colours)

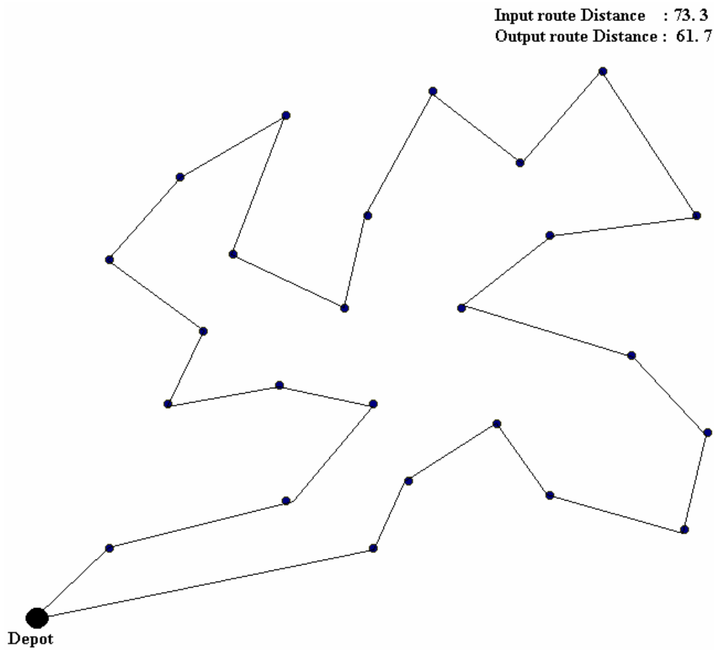
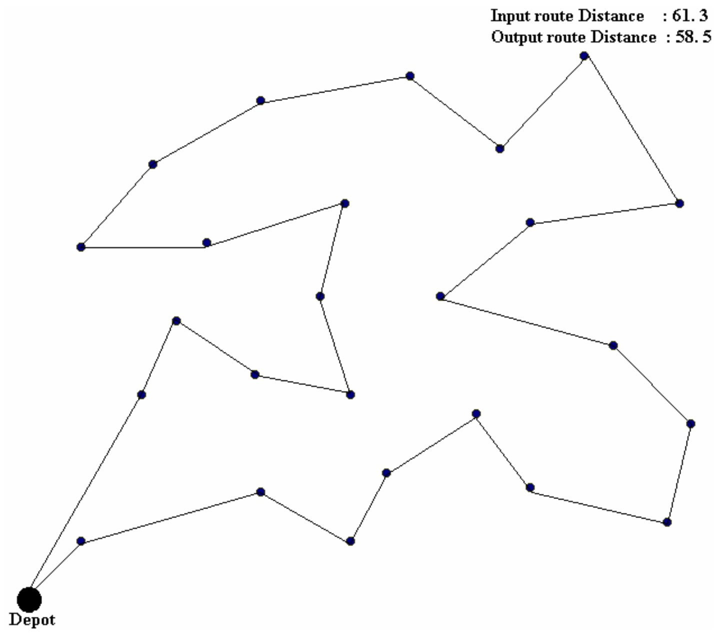


Figure 7 Output obtained for proposed heuristic input sequence (see online version for colours)



10 Conclusions

The results shows that, by directly applying the shrink wrap algorithm input for SA, the distance is 61.7, where as the by using directly the nearest neighbour heuristic, the distance is 59.5. By combining both the shrink wrap and unwrapping heuristic, the result is 58.5, which is still better than the input mode 2. So, with the proposed heuristic, the optimisation is increased by 1.7%. Since, SA do not guarantee optimal solutions, the problem may still be analysed for further optimisation.

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