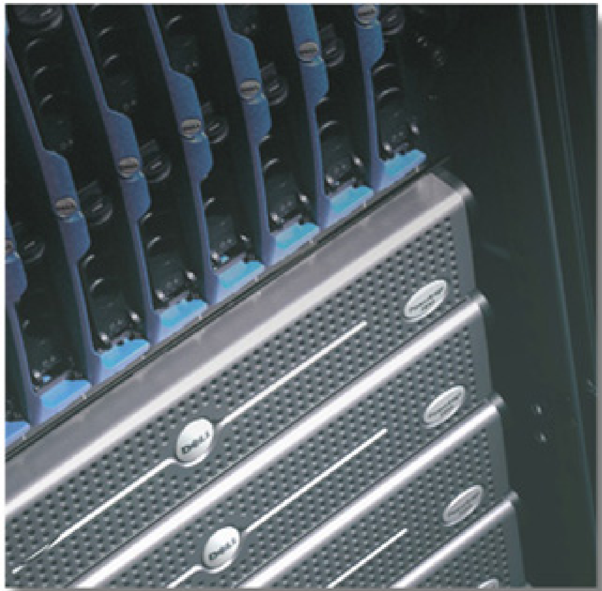

Forecasting of Short Life Cycle Products and Parts using Nonlinear Optimization

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Agenda

- Challenges in forecasting of short life cycle products.
- Bass diffusion model for new products long range forecasting
- Bass Model Variations
- Kurawarwala’s model for forecasting of short life cycle products with seasonal characteristics
- Bass Model Application for Product transition problem
- New products diffusion under Supply Constraints
- Conclusions

Introduction to Dell Peripherals Forecasting

- Dell's direct model enables customers to configure systems based on their needs and place orders accordingly.
- For better customer experience, it is important to keep required raw materials in inventory for fulfilling orders at the earliest possible time.
- Procurement lead times for peripherals/components are long.
- Component prices generally drop over a period of time and are also volatile.
- Forecasting plays an important role in procurement and inventory decisions
- Bad forecasts result in higher costs due to excess inventory costs, shortage costs, expedition costs etc.

Factors Influencing Forecast Accuracy

Factor	Challenges in Forecast Accuracy
Life Cycle Characteristics	Short life cycle: Not enough history for using statistical forecasting methods
Peripherals prices	Rapid drop in prices, potentially every week
Technology status	High End to low end have influence on sales
Sales channels	Each sales channel may have different seasonal characteristics
Promotions	May increase sales for promotion products and may cannibalize sales for other products
Competitors strategies	Influence on end-systems prices and promotions
Executive directions	In terms of growth and profitability may influence pricing strategies and there by demand
Market shortages and Demand shaping decisions	May alter the peripherals demand
Association of peripherals with platforms	Many to many association which in turn influences demand of peripherals along with demand of its associated platforms

Forecasting Problem for Short Life Cycle Products

- Peripherals Forecasting Needs at Dell
  - Need to have a long term forecast (typically one year) for short life cycle peripheral
  - Preferable to forecast the entire life cycle with a limited or no historical sales data for a peripheral
- What is Available
  - Historical sales data of old peripherals
  - Probably most old peripherals may have reached End-Of-Life (EOL)
- Challenge
  - Can we extract any possible forecasting characteristics from old peripherals data and use it in forecasting for new peripherals?

Broad Ideas for Addressing Forecasting Problem

- A model should be able to capture the entire life cycle characteristics of products
- Capture the different characteristics from historical sales data for old peripherals
- Group the peripherals based on a similarity characteristic. Here we assume seasonal characteristics are common for all peripherals in a group
- Compute the seasonality characteristic at the group level
- Use the seasonality characteristic of the group and experts' knowledge for arriving at different forecasting scenarios for a new peripheral

Bass Diffusion Model

- Bass model provides theoretical framework for with long range forecasting of new product sales based on theory of timing of adoption
- Two key estimates from Bass Model: 1) Sales over time 2) Peak time sales
- Two classes of adopters for new products: 1) Innovators who adopt innovation independently and 2) Imitators whose timing of adoption is influenced by pressures of social system
- T*he probability that an initial purchase will be made at time T given that no purchase has been made is a linear function of the number of previous buyers, i.e.* $P(T) = p + (q/m)*Y(T)$, where
  - $Y(T)$ is the number of previous buyers, p is coefficient of innovators, q is coefficient of imitators and m is market size
- Let $f(T)$ is the likelihood purchase at time T and $F(T)$ is cumulative density function of purchase till time T, then

Bass Theory (Contd.)

$$F(T)=\int_0^T f(T)dt \qquad \& \quad F(0)=0$$

Let $S(T)$ refers to sales in time T. *Then*

$$Y(T)=\int_0^T S(T)dt = m\int_0^T f(T)dt = m*F(T)$$

So $$\frac{f(T)}{1-F(T)} = P(T) = p+(q/m)*Y(T) = p+qF(T)$$

Sales at T is

$$S(T) = mf(T) = P(T)\left(m-Y(T)\right) =$$

$$=\left[p+q\int_0^T S(T)\,dt/m\right]\left[m-\int_0^T S(T)dt\right]$$

Expanding this product, we get

$$S(T)= pm+(q-p)Y(T)-q/m\left[\mathrm{Y(T)}\right]^2$$

Bass Theory (Contd.)

As a differential equation, the above is

$$\frac{dY(t)}{dt} = \left(p + (q/m)Y(t)\right)\left(m - Y(t)\right)$$

Solution to above equation, given $Y(0) = 0$ *is*

$$Y(T) = m * \left[\frac{1 - e^{-(p+q)T}}{1 + {}^{q}\!/_{p}\, e^{-(p+q)T}} \right]$$

$$S(T) = m * \frac{\left[(p+q)^2 / p\right] e^{-(p+q)T}}{\left[1 + \frac{q}{p} e^{-(p+q)T}\right]^2}$$

Peak time sales is when $S(T)$ is maximum

$$T^* = \frac{1}{p+q} \ln\left(\frac{q}{p}\right)$$

Bass Model Variations

- Generalized Bass model to capture the effects of decision variables such as price and advertisement on new product sales.
- Kurwarwala's model for capturing the seasonal effects on product diffusion
- Sales of successive generations of products
- Bass model for sales during product transition (i.e. from a product that is reaching EOL to a new product that is replacing it)
- New product diffusion under supply constraints

Generalized Bass Model

- Provides framework for understanding the impact of decision variables such as price and advertisement.

$$\frac{f(T)}{1-F(T)} = \left(p + qF(T)\right)x(T)$$

For price and advertisement decision variables

$$x(T) = \left(1 + a_1 \Delta p + a_2 \Delta A\right)$$

Here Δp is % decline in price and

ΔA is % increase in advertisement

Kurawarwala's Extension of Bass Model

- Kurawarwala's model is an extension of Bass Model that also takes into account the seasonal characteristics of a product
- Abbas Kurawarwala developed models in early 90's to address Dell's challenges of forecasting of products with short life cycle
- The model provides a way of determining common seasonal characteristics from historical data of old products and use it in forecasting of a new product.
- From historical data of a group of products it estimates for each product the influence of innovators, imitators, market size and common seasonal indices so that the error between estimates and actual data is minimum
- Using the group's common seasonal indices and knowledge of peak sales timings, the model estimates reasonably well the long range forecast of a new product

Kurawarwala's model (contd.)

$$\frac{dN_t}{dt} = \left(p + (q/m)N_t\right)(m - N_t)\alpha_t$$

Where N_t is the cumulative demand till time t,

$p, q, \& \, m$ are similar to Bass model parameters

α_t is the seasonal influence parameter

$$N_0 = 0$$

$$N_t = m * \left[\frac{1 - e^{-(p+q)\int_0^t \alpha_\tau d\tau}}{1 + \frac{q}{p} e^{-(p+q)\int_0^t \alpha_\tau d\tau}} \right]$$

Kurawarwala's Model (contd.)

When $\alpha_t = 1$ for every time period,

then above equation is same as Bass model

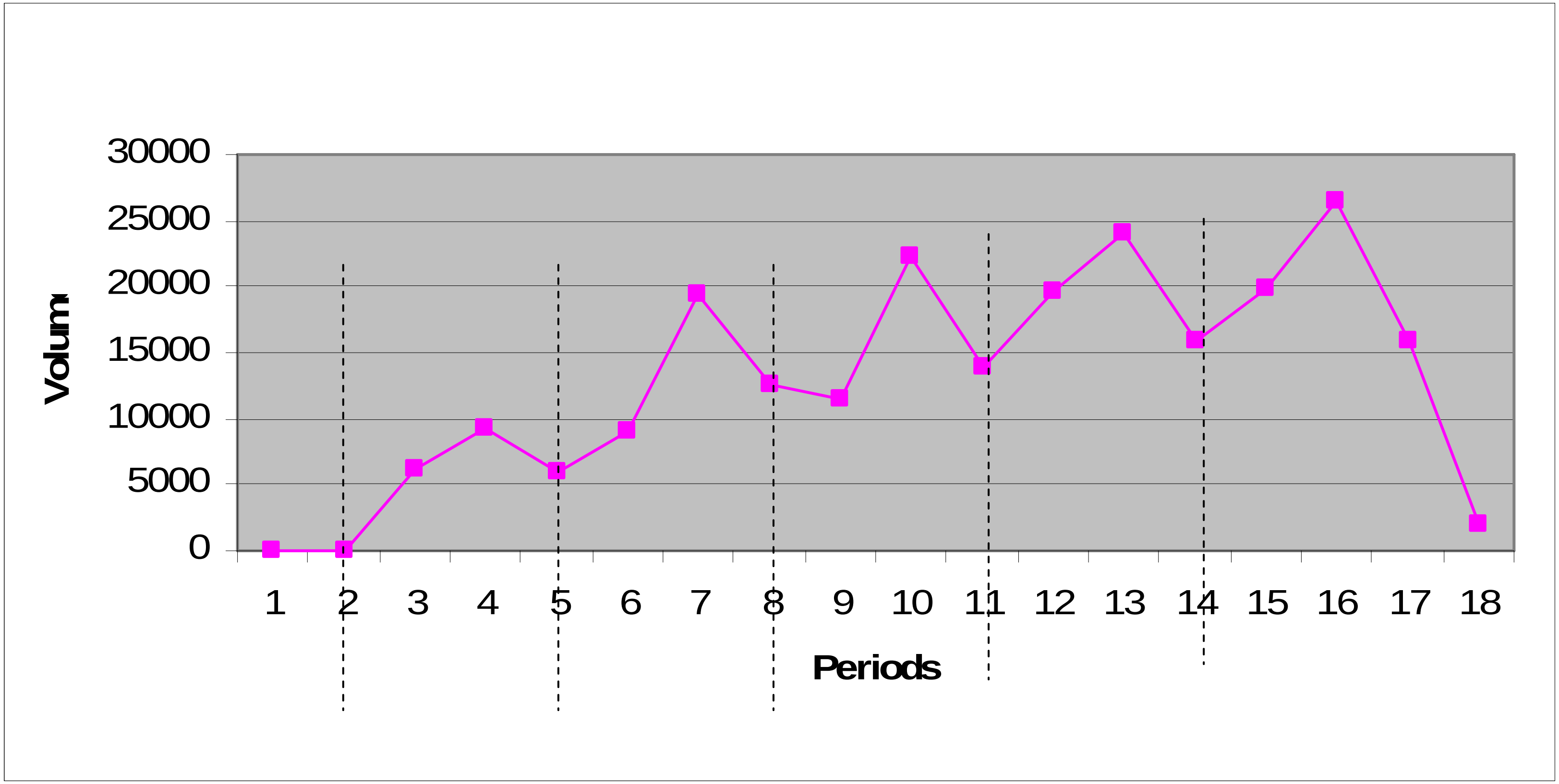
α_t rescales the time axis of the Bass model

In Dell, the seasonal pattern repeats annually.

The peak time sales is given by equation

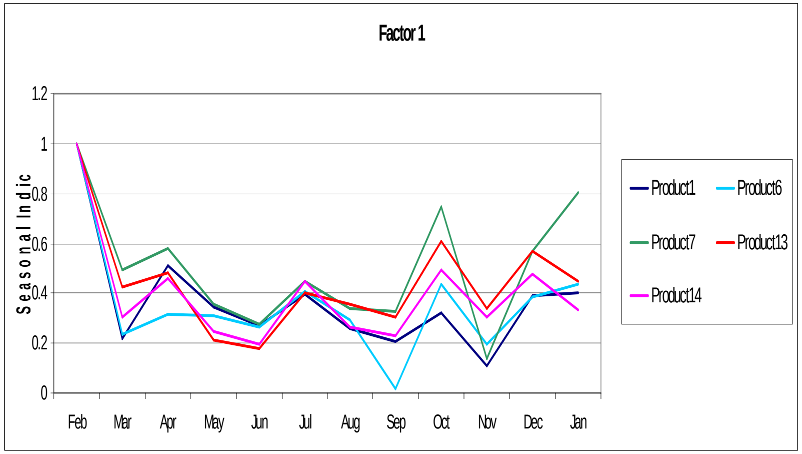
$$\int_0^{t^*} \alpha_t = \frac{1}{p+q} \ln\left(\frac{q}{p}\right)$$

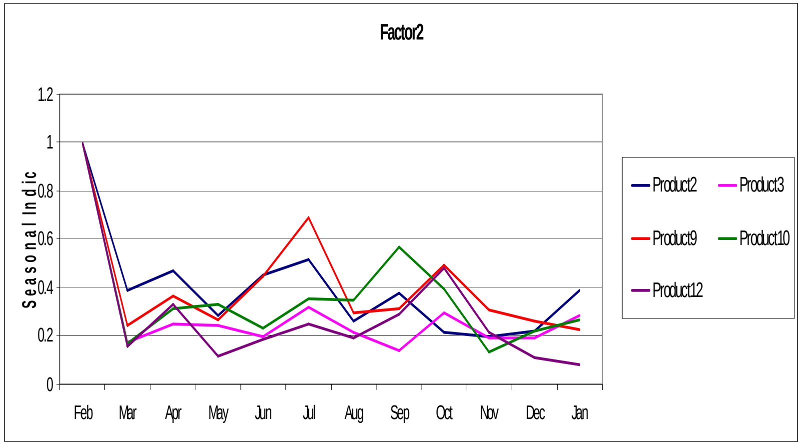
Example of Seasonal Characteristics of a Peripheral

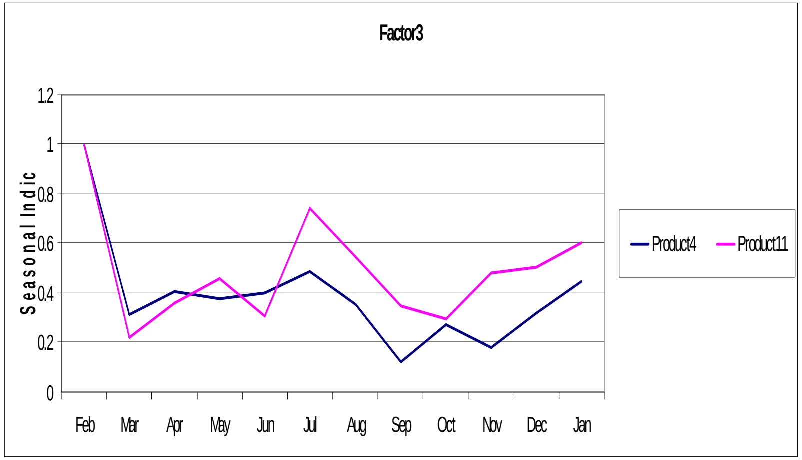


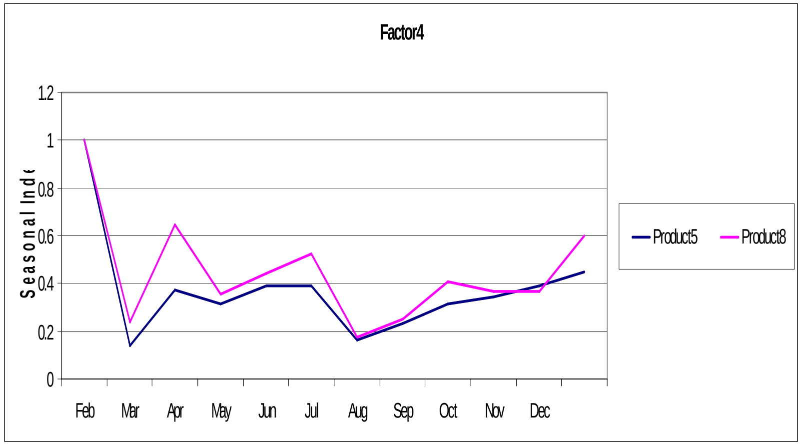
Seasonality is repeating every quarter

Grouping products with common seasonal pattern using Clustering Analysis









Extracting Information From Old Products Historical Data

Basic Assumption : Old products in a group share similar seasonal pattern

$n =$ number of products

$T =$ last month in the horizon

$i = 1 \ldots n, \qquad t = 1 \ldots T$

$x_{it} =$ Demand for product i in time t

$\alpha_t =$ Seasonal index for month t

$s_i, l_i =$ first and last months of $+$ve demand forProduct i

$p_i, q_i, m_i =$ Bass parameters for product i

$$N_{it} = m_i * \left[\frac{1 - e^{-(p_i + q_i)\sum_{\tau=0}^{t} \alpha_t}}{1 + \frac{q_i}{p_i} e^{-(p_i + q_i)\sum_{\tau=0}^{t} \alpha_t}} \right]$$

The nonlinear least squares estimation problem is

$$\min Z = \sum_{i=1}^{n} \left\{ \sum_{t=s_i}^{l_i} \left[x_{it} - \left(N_{it} - N_{i,t-1} \right) \right]^2 \right\}$$

subject to

$$\alpha_1 = 1.0, \quad \alpha_t \geq 0 \quad \forall t$$

$$\alpha_t = \alpha_{t-12} \quad \forall t = 13 \cdots T$$

$$p_i \geq 0, q_i \geq 0, m_i \geq 0 \quad \forall i$$

Estimating Parameters Initial Values for NLP

Estimating initial values of α_t

$$\alpha_t = \frac{\bar{x}_t}{\bar{x}}, \text{ where } \bar{x}_t \text{ is the period average } \&$$

$$\bar{x} \text{ is the grand average}$$

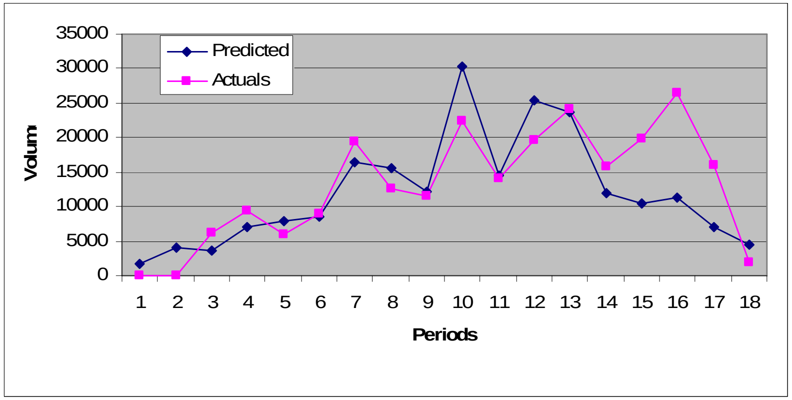
Estimating initial values of $p \,\&\, q$ *based on initial* α_t

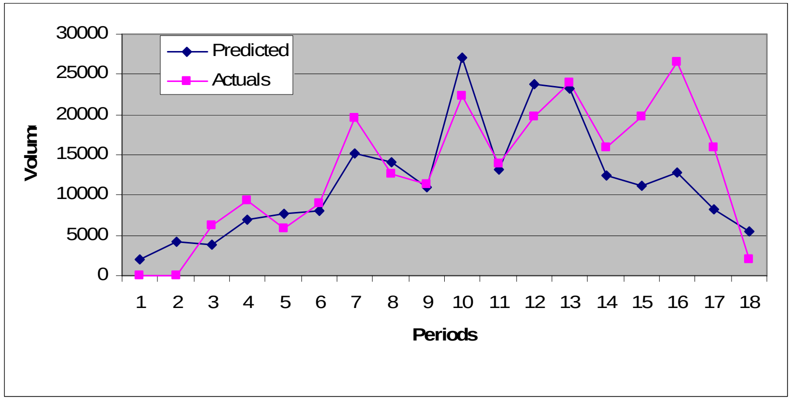
$$S_t = \left(pm + (q-p)N_t - (q/m)N_t^2\right)\sum_{\tau=0}^{t}\alpha_\tau$$

a: pm b: $(q-p)$ c: (q/m)

With known S_t and N_t values, the above is a linear regression problem based on which *p*, *q* & *m* can be estimated

New Products Forecasting Scenarios based on Experts Knowledge





Scenario 1

•Expected Demand in period 8 is in between 60,000 and 65,000.

Scenario 2

•Expected Demand in period 13 is in between 150,000 and 160,000.

In both scenarios first three months actual sales data is used and total market is assumed to be in between 230,000 and 250,000

Bass Model Application for Product Transition

- Product transition occurs when a new product is replacing an EOL product and the transition time is when both products are active.
- Product transition problem deals with predicting how the sales of an EOL product is declining and new product sales are growing during transition time.
- We consider peak market share as the market share of the product when it is at the end of growth phase
- Based on the market share (with respect to Line of Business) of the EOL product during its growth phase, we estimate the parameters of Bass diffusion model.
- We consider that peak market share is shared by both product during transition time
- For new product, the market share is same as that of growth phase of the EOL product and the declining share of the EOL product is equal to difference between peak market share and new product's market share.

New Product Diffusion under Supply Constraints

- Under supply constraints, unmet demand for a product in a time period can result in backlog demand and some cancellations in the next period .

$$D_t = \text{Cumulative demand till time } t$$

$$S_t = \text{Cumulative sales (fulfilled) till time } t$$

$$W_t = \text{Waiting list of customers in time } t$$

$$L_t = \text{Lost customers in time } t$$

$$D_t = S_t + W_t + L_t$$

Product diffusion under above conditions

$$\frac{dD_t}{dt} = \left(p + (q/m)S_t\right)\left(m - D_t\right)$$

Conclusions

- Bass model is a very versatile model for long range forecasting of new products.
- It can be easily modified to include various business needs in new product forecasting
- Illustrated interesting modifications to model for:
  - Long range forecasting of short life cycle products with seasonal characteristics
  - Product transitions
  - New product diffusion under supply constraints