Decentralized supply chain formation using an incentive compatible mechanism

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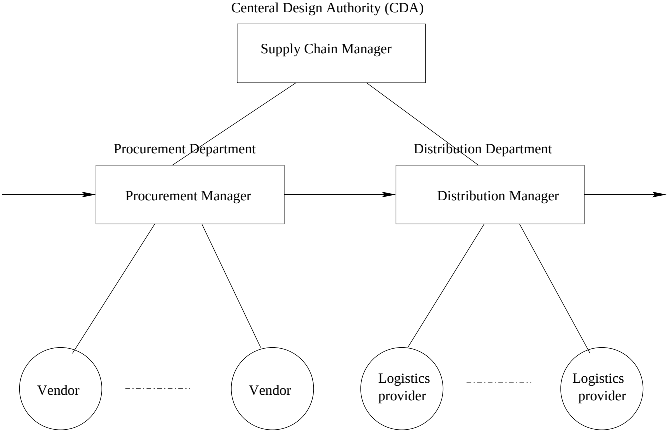
Symposium on “Optimization in Supply Chains”

IIT Bombay, Oct 27, 2007

Outline

A two echelon supply chain



An example

- Procurement manager chooses a vendor. Similarly, logistics manager.
- Vendors give quotes as:

Service Provider	μ (days)	σ (days)	Cost
Provider 1	3	0.5	2500
Provider 2	3	0.75	1500
Provider 3	3	1.0	1250
Provider 4	4	1.0	1000
Provider 5	4	1.25	750
Provider 6	5	1.50	500

Table: Delivery quality and costs offered by six logistics providers to the distribution manager

- Procurement and distribution managers give cost curves to supply chain manager
- Supply chain manager seeks a cost-optimal combination that meets QoS levels
- Echelon managers seek to maximize profits of their units (perhaps, independent)
- Quoted cost curves need not be actual ones
- Can have a strategic play, inducing a game

- Supply chain manager (Central Design Authority) lacks actual information that echelon managers have
- Aim: Cost-optimal chain formation with incomplete (decentralized) information that should satisfy specified QoS levels
- We stick to a single echelon framework
- A two-step procedure:
  1. Design an incentive compatible protocol (mechanism) to elicit true costs
  2. Solve an appropriate constrained optimization problem with these values

Mean variance allocation problem

- Let n-echelons in a linear network have delivery times X_i, Independent normal rvs; means μ_i and standard deviation σ_i
- End-to-end delivery time, Y is normal with mean $\mu = \sum_i^n \mu_i$ and standard deviation $\sigma = \sum_i^n \sigma_i$
- Suppose τ is target date and T is tolerance allowed; CDA aims for a delivery within $\tau \pm T$ days
- Supply chain process capability indices, C_p and C_{pk} are:

$$C_p = \frac{U-L}{6\sigma} = \frac{T}{3\sigma}$$

$$C_{pk} = \frac{\min(U-\mu, \mu-L)}{3\sigma}$$

CDA knows these:

1. The delivery window $(\tau - T, \tau + T)$
2. Lower bounds of C_p and C_{pk} as $C_p \geq p$ and $C_{pk} \geq q$.
3. Lower bounds $\underline{\mu_i}$ and $\underline{\sigma_i}$ on the mean μ_i and standard deviation σ_i, respectively, of stage i ($i = 1, \ldots, n$). Similarly, upper bounds $\overline{\mu_i}$ and $\overline{\sigma_i}$.
4. Delivery cost function $b_i(\mu_i, \sigma_i)$ per unit order submitted by the manager of echelon i.

Mean variance problem is

$$\text{minimize} \quad \sum_{i=1}^{n} b_i(\mu_i, \sigma_i)$$

subject to:

$$C_p \geq p$$

$$C_{pk} \geq q$$

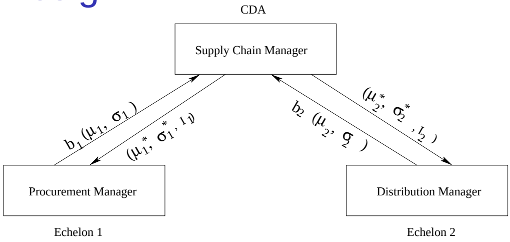
$$\tau - T \leq \sum_{1}^{n} \mu_i \leq \tau + T$$

$$\underline{\mu_i} \leq \mu_i \leq \overline{\mu_i}; \quad \underline{\sigma_i} \leq \sigma_i \leq \overline{\sigma_i}; \quad i \in N$$

Informal example

- In an (English) auction the winner just needs to bid incrementally more than the second highest bidder
- However, auctioneer can not know winner's willingness to pay (true valuation)
- Suppose auctioneer conducts sealed bid *second* price auction (Vickery auctions)
- Here, winner gets the item at the bid-price of second highest bidder
- Under some more conditions winner will now give true valuation
- Can be interpreted as a Mechanism where the auctioneer is paying an incentive to winner, the difference between highest bid and second highest bid

Mechanism Design



- Assumption: cost curves for i^{th} echelon are:

$$c_i(\mu_i, \sigma_i) = a_{i0} + a_{i1}\mu_i + a_{i2}\sigma_i + a_{i3}\mu_i\sigma_i + a_{i4}\sigma_i^2$$

- Private information of i^{th} manager is 5-tuple of coefficients $(a_{i0}, a_{i1}, a_{i2}, a_{i3}, a_{i4})$.
- Echelon managers report $b_1(.), \ldots, b_n(.)$ as

$$b_i(\mu_i, \sigma_i) = \hat{a}_{i0} + \hat{a}_{i1}\mu_i + \hat{a}_{i2}\sigma_i + \hat{a}_{i3}\mu_i\sigma_i + \hat{a}_{i4}\sigma_i^2; \quad i = 1, \ldots, n$$

- A Mechanism Design model gives true values of these costs against incentives.
- CDA is viewed as a social planner and echelon managers as agents.

Notation

N	$=$	$\{0, 1, \dots, n\}$, the set of players 0 corresponds to the CDA while $1, \dots, n$ correspond to the echelon managers
θ_i	$=$	$(a_{i0}, a_{i1}, a_{i2}, a_{i3}, a_{i4})$ is the private information (type) of player i
$\hat{\theta}_i$	$=$	$(\hat{a}_{i0}, \hat{a}_{i1}, \hat{a}_{i2}, \hat{a}_{i3}, \hat{a}_{i4})$ is the reported type of player i
c_i	$=$	True cost function (actual type) of player i; $c_i(\mu_i, \sigma_i) = a_{i0} + a_{i1}\mu_i + a_{i2}\sigma_i + a_{i3}\mu_i\sigma_i + a_{i4}\sigma_i^2$
b_i	$=$	Reported cost function (reported type) of player i; $b_i(\mu_i, \sigma_i) = \hat{a}_{i0} + \hat{a}_{i1}\mu_i + \hat{a}_{i2}\sigma_i + \hat{a}_{i3}\mu_i\sigma_i + \hat{a}_{i4}\sigma_i^2$
Θ_i	$=$	Set of all possible types of player i
Θ	$=$	$\Theta_0 \times \Theta_1 \times \Theta_2 \times \dots \times \Theta_n$; $\theta = (\theta_0, \theta_1, \dots, \theta_n) \in \Theta$
Θ_{-i}	$=$	$\Theta_0 \times \dots \times \Theta_{i-1} \times \Theta_{i+1} \times \dots \times \Theta_n$; $\theta_{-i} = (\theta_0, \dots, \theta_{i-1}, \theta_{i+1}, \dots, \theta_n) \in \Theta_{-i}$

Assumptions

1. $\Theta_0 = \{\theta_0\}$; that is type set of CDA is a singleton. Needed for Dominant strategy incentive compatible mechanism but not for weaker Bayesian incentive compatible mechanism.
2. $\mu_i \in [\underline{\mu_i}, \overline{\mu}_i]$, $i = 0, 1, \cdots, n$
3. $\sigma_i \in [\underline{\sigma_i}, \overline{\sigma}_i]$
4. Actual costs are

$$c_i(\mu_i, \sigma_i) = a_{i0} + a_{i1}\mu_i + a_{i2}\sigma_i + a_{i3}\mu_i\sigma_i + a_{i4}\sigma_i^2 \quad \forall \mu_i \in [\underline{\mu_i}, \overline{\mu}_i] \quad \forall \sigma_i \in [\underline{\sigma_i}, \overline{\sigma}_i]$$

5. Coefficients $a_{i0}, a_{i1}, a_{i2}, a_{i3}$, and a_{i4} come from some given intervals:
   $a_{ij} \in \left[\underline{a}_{ij}, \overline{a}_{ij}\right]$ for $j = 0, 1, 2, 3, 4$.
6. These give type sets: Θ_i as

$$[\underline{a}_{i0}, \overline{a}_{i0}] \times [\underline{a}_{i1}, \overline{a}_{i1}] \times [\underline{a}_{i2}, \overline{a}_{i2}] \times [\underline{a}_{i3}, \overline{a}_{i3}] \times [\underline{a}_{i4}, \overline{a}_{i4}]$$

   Θ is a compact set in $\mathbb{R}^5$.

- Outcome set X:
  - Vector $x = (k, l_0, l_1, \cdots, l_n)$ where
  - $k = (\mu_0, \sigma_0, \mu_1, \sigma_1, \cdots, \mu_n, \sigma_n)$ is called allocation (project choice) vector and
  - $l_0, l_1, \cdots, l_n$ are money transfers (payments) to CDA, manager 1, ...

- μ_i and σ_i are the assigned mean and standard deviation to the echelon i. Also,

$$\mu_0 = \mu_1 + \ldots + \mu_n$$

$$\sigma_0^2 = \sigma_1^2 + \ldots + \sigma_n^2$$

  For $i = 1, \ldots, n$, l_i is the total budget sanctioned by the CDA for the manager of echelon i.
  l_0 is the total budget available with the CDA.

- The set of feasible outcomes is

$$X = \left\{(\mu_i, \sigma_i, I_i)_{i=0,1,\ldots,n} | \mu_i \in [\underline{\mu_i}, \overline{\mu}_i] \quad \sigma_i \in [\underline{\sigma_i}, \overline{\sigma}_i], I_i \in \mathbb{R}\right\}$$

The set of project allocations $\{k\}$s is K (and is compact).

- Valuations:
Let the value of allocation k for player i be $v_i(k, \theta_i)$ when the type set is θ_i. Define,

$$\begin{aligned} v_i(\mu_0, \sigma_0, \mu_1, \sigma_1, \ldots, \mu_n, \sigma_n; \theta_i) &= -c_i(\mu_i, \sigma_i) \\ &= -(a_{i0} + a_{i1}\mu_i + a_{i2}\sigma_i + a_{i3}\mu_i\sigma_i + a_{i4}\sigma_i^2) \end{aligned}$$

- Players' Utility:
  The i^{th} player's utility $u_i(\cdot) : X \times \Theta_i$ to $\mathbb{R}$ is taken as

$$u_i(k, I_0, I_1, \ldots, I_n; \theta_i) = v_i(k, \theta_i) + I_i + E_i$$

  where E_i is an initial endowment with player i ($i = 0, 1, \ldots, n$) and could be taken as zeroes.
  This gives the quasi-linear mechanism design framework.

- Social Choice function $f(\cdot) : \Theta$ to $\mathbb{R}$:
  We take this as

$$f(\theta) = (\mu_i(\theta), \sigma_i(\theta), I_i(\theta))_{i=0,1,\ldots,n}, \ \forall\, \theta \in \Theta$$

Ex-post Efficiency

- A SCF $f(\cdot)$ is called ex-post efficient if$\forall \theta \in \Theta$, the outcome $f(\theta)$ is such that there does not exist any $x \in X$ such that

$$u_i(x, \theta_i) \geq u_i(f(\theta), \theta_i) \quad \forall i \in N$$

$$u_i(x, \theta_i) > u_i(f(\theta), \theta_i) \text{ for } \quad \text{some } \quad i \in N$$

- In an ex-post efficient supply chain formation, payoffs are such Pareto optimal—utility of a player is improved at the expense of at least one other players' utility.
- Fact: In a quasi-linear environment, ex-post efficiency is equivalent to simultaneously having Allocative efficiency (AE) and Budget balance (BB).

Allocative efficiency (AE)

- A SCF $f(.) = (k(.), I_0(.), I_1(.), \ldots, I_n(.))$ is AE over all the echelon managers if $\forall \theta \in \Theta, k(.)$ satisfies

$$\sum_{i=1}^{n} v_i(k(\theta),\theta_i) \geq \sum_{i=1}^{n} v_i(k,\theta_i) \; \forall k \in K$$

- Each allocation $k \in K$ maximizes the total valuations of echelon managers.
- Since, valuation of CDA is sum of valuations of managers, we then have

$$\sum_{i=0}^{n} v_i(k(\theta),\theta_i) \geq \sum_{i=0}^{n} v_i(k,\theta_i) \; \forall k \in K$$

  Now, SCF is AE over all players in the game.
- Such an allocation can be obtained by solution of MVA problem:

$$f(\theta) = (\mu_i^*(\theta), \sigma_i^*(\theta), I_i(\theta))_{i=0,1,\ldots,n}, \;\; \forall \; \theta \in \Theta$$

  where $(\mu_i^*(\theta), \sigma_i^*(\theta))_{i=0,1,\ldots,n}$ is the solution of the earlier MVA problem.

Budget Balance (BB)

- A SCF $f(.) = (k(.), I_0(.), I_1(.), \ldots, I_n(.))$ is said to be budget balanced if $\forall \theta \in \Theta$, we have

$$\sum_{i=0}^{n} I_i(\theta) = 0$$

- Supply chain is then formed with no deficit or surplus by distributing budget among all players.

Aim: A formation that is AE, BB that also induces truth revelation from echelon managers.

Dominant Strategy Incentive Compatible Mechanism (DSIC)

- $(\mu_i^*(\theta), \sigma_i^*(\theta))_{i=0,1,\dots,n}$ make SCF $f(\theta)$ is allocatively efficient
  We choose budgets $(I_i(\theta))_{i=0,1,\dots,n}$ so that it is also possible to have the SCF $f(.)$ dominant strategy incentive compatible *i.e.* echelon managers will report true values.

- Fact Groves mechanism are both AE and DSIC.

$$I_i(\theta) = \alpha_i(\theta_{-i}) - \sum_{j \neq i} b_j(\mu_i^*(\theta), \sigma_i^*(\theta)) \ \forall \theta \in \Theta$$

where $(\mu_0^*(\theta), \dots, \mu_n^*(\theta), \sigma_0^*(\theta), \dots, \sigma_n^*(\theta))$ is the optimal solution of the MVA problem.
For $i = 0, 1, 2, \dots, n$, $\alpha_i(\theta_{-i})$ is any arbitrary function from Θ_{-i} to $\mathbb{R}$.

- Fact AE, BB and DSIC may not be simultaneously possible if cost functions are sufficiently rich.
- Fact Above is possible if one agent's type set is singleton.
- Choose α_i's so that $\sum_0^n l_i(\theta) = 0 \quad \forall \quad \theta \in \Theta$. Take,

$$\alpha_j(\theta_{-j}) = \begin{cases} \alpha_j(\theta_{-j}) & : \quad j \neq i \\ -\sum_{r \neq i} \alpha_r(\theta_{-r}) - (n) \sum_{r=0}^{n} v_r(k^*(\theta), \theta_r) & : \quad j = i \end{cases}$$

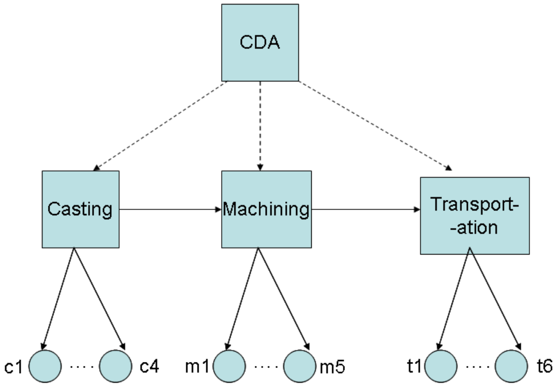
- To summarize:
  - Cost-optimal solution that also meets QoS requirements (via AE)
  - Has Budget balance (BB)
  - Induces truth revelation by echelon managers (DSIC)
- Ensures that each manager's action is optimal irrespective of what others do
- Payments tend to be high

Bayesian Incentive Compatible solution (BIC)

- Assume that type sets are statistically independent.
- The dAGVA theorem (d'Aspremont and Gérard-Varet and Arrow) suggests the payments to be

$$I_i(\theta_i, \theta_{-i}) \quad = \beta_i(\theta_{-i}) \quad + E_{\tilde{\theta}_{-i}}[\sum_{j \neq i} v_j(k^*(\theta_i, \tilde{\theta}_{-i}), \tilde{\theta}_j)]$$

  where $\beta_i : \Theta_{-i} \to \mathbb{R}$ is any arbitrary function.
- Can now choose to ensure Budget balance (BB).
- The type set of CDA need not be singleton
- Numerical examples show that BIC payments are lower than those of DSIC.


CDA
Casting
Machining
Transport-
-ation
c1
c4
m1
m5
t1
t6

(Data is skipped)

Echelon i	Payments for SCF-DSIC	Payments for SCF-BIC
1	207.00	80.00
2	219.80	83.00
3	160.80	68.30

Table: Each agent believes that other agents equally like to be truthful or untruthful

Echelon i	Payments for SCF-DSIC	Payments for SCF-BIC
1	207.00	159.50
2	219.80	166.00
3	160.80	136.50

Table: Each agent believes that each other agent is completely truthful

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