Autumn Semester **September 14, 2020**

IE 605: Engineering Statistics

Tutorial 2

Exercise 1 *(a) Suppose that an event E is independent of itself. Show that either $P(E) = 0$ or $P(E) = 1$.*
(b) Events A and B have probabilities $P(A) = 0.3$ and $P(B) = 0.4$. What is $P(A \cup B)$ if A and B are independent? What is $P(A \cup B)$ if A and B are mutually exclusive?
(c) Now suppose that $P(A) = 0.6$ and $P(B) = 0.8$. In this case, could the events A and B be independent? Could they be mutually exclusive?

Exercise 2 *(a) Let X represent the lifetime, rounded up to an integer number of years, of a certain car battery. Suppose that the pmf of X is given by $p_X(k) = 0.2$ if $3 \leq k \leq 7$ and $p_X(k) = 0$ otherwise.*
(i) Find the probability, $P\{X > 3\}$, that a three year old battery is still working.
(ii) Given that the battery is still working after five years, what is the conditional probability that the battery will still be working three years later? (i.e. what is $P(X > 8|X > 5)$)?
(b) A certain Illini basketball player shoots the ball repeatedly from half court during practice. Each shot is a success with probability p and a miss with probability $1 - p$, independently of the outcomes of previous shots. Let Y denote the number of shots required for the first success.
(i) Express the probability that she needs more than three shots for a success, $P\{Y > 3\}$, in terms of p.
(ii) Given that she already missed the first five shots, what is the conditional probability that she will need more than three additional shots for a success? (i.e. what is P $(Y > 8|Y > 5)$)?
(iii) What type of probability distribution does Y have?

Exercise 3 *Suppose each corner of a cube is colored blue, independently of the other corners, with some probability p. Let B denote the event that at least one face of the cube has all four corners colored blue.*

1. *Find the conditional probability of B given that exactly five corners of the cube are colored blue.*

2. *Find P(B), the unconditional probability of B*

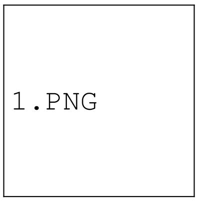
Exercise 4 *Which of the following are valid CDF's? For each that is not valid, state at least one reason why. For each that is valid, find $P(X^2 > 5)$.*

1.
$$F(x) = \begin{cases} e^{-x^2}/4 & \text{if} \quad x < 0 \\ 1 - e^{-x^2}/4 & \text{if} \quad x \geq 0 \end{cases} \tag{1}$$

2.
$$F(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ 0.5 + e^{-x} & \text{if} \quad 0 \leq x < 3 \\ 1 & \text{if} \quad x \geq 3 \end{cases} \tag{2}$$

3.
$$F(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ 0.5 + x/20 & \text{if} \quad 0 \leq x \leq 10 \\ 1 & \text{if} \quad x \geq 10 \end{cases} \tag{3}$$

Exercise 5 *Let X have the CDF shown.*



1. *Find* $P(X \leq 0.8)$.
2. *Find* $E(X)$.
3. *Find* $Var(x)$.

Exercise 6 *Let* X *is a random variable with probability density function*

$$f_X(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find $P(X \geq 0.4 | X \leq 0.8)$.

Exercise 7 *Suppose five fair coins are tossed. Let* E *be the event that all coins land heads. Define a random variable* I_E

$$I_E = \begin{cases} 1 & \text{if } E \text{ occurs} \\ 0 & \text{if } E^c \text{ occurs} \end{cases}$$

For what outcomes in the original sample space does I_E *equals 1 ? what is* $P\{I_E = 1\}$

Exercise 8 *Suppose a coin having probability* 0.7 *of coming up heads is tossed three times. Let* X *denote the number of heads that appear in the three tosses. Determine the probability mass function of* X.

Exercise 9 *Suppose the distribution function of X is given by*

$$F(b) = \begin{cases} 0, & b < 0 \\ \frac{1}{2}, & 0 \leq b < 1 \\ 1, & 1 \leq b < \infty \end{cases}$$

What is the probability mass function of X?

Exercise 10 *A coin having probability p of coming up heads is successively flipped until the rth head appears. Argue that X, the number of flips required, will be n, $n \geq r$, with probability*

$$P(X = n) = \tbinom{n-1}{r-1} p^r (1-p)^{n-r}, n \geq r$$

Exercise 11 *The probability mass function of X is given by*

$$p(k) = \tbinom{r+k-1}{r-1} p^r (1-p)^k, k = 0, 1, \ldots$$

Give a possible interpretation of the random variable X.

Exercise 12 *If the density function of X equals*

$$f(x) = \begin{cases} ce^{-2x} & \text{if } 0 \leq x < \infty \\ 0 & \text{if } x < 0 \end{cases}$$

find c. What is the value of $P\{X > 2\}$?