An analysis of operations, mode choice, pricing and network economics of container movement

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ABSTRACT
This paper describes the characteristic features of multi-modal transport and containerized movement of goods. It outlines some elements of an integrated logistics and pricing strategy and the various aspects involved in competition between different modes of transport on a transport network. It also describes the various factors affecting mode choice. Transport operations have been modeled taking into account the locational and temporal mismatches in supply and demand of transport requirements. Pure locational imbalance of demand necessitates empty movements of vehicles while pure temporal imbalance of demand calls for waiting at locations. Both these imbalances lead to additional costs. This paper seeks to capture both these elements of cost along with the normal operating costs and formulates a pricing strategy for competing operators based on the game-theoretic notion of equilibrium. Under competition, an operator's profit depends not only on his actions but also on the actions of the competitors. The demand is shared based on the fare charged, the frequency offered and a factor accounting for the quality of service. A profit maximizing strategy for each of the competing operators has been developed. The application of this model is intended for modeling of containerized transport on a particular sector in India.

Keywords
Transport Networks, Containerization, Mode Choice, Transport Pricing, Operating Policy, Competition.

1. INTRODUCTION
The basic intent of this paper is to model transport operations of containerized freight traffic on a network. A motivating scenario is that of containerized movement of import and export goods to and from India. In at least one major stream of such movements (from the Delhi area to the one in Mumbai), there are competing modes of movement, viz. road and rail. This paper attempts to set up the framework to analyze operating strategies, pricing decisions and the resulting market shares in such a competitive environment.

Nodes on a transport network denote the various locations and demand for transport (in terms of tons/hr) exists between node pairs. Network-wide operations on a fully connected transport network have been considered.

In section 3, the operating strategy of transporters has been modeled taking into account the locational as well as temporal mismatches in supply and demand of transport requirements. A pure locational imbalance of demand for transport creates some guaranteed empty movements, which have to be costed. A pure temporal imbalance of demand for transport calls for excess capacity of transport vehicles and waiting at locations. Both these elements of cost along with normal operating costs, in a random demand environment have been captured.

Further, competition between different modes is considered in section 4. Modal choice characteristics have been studied. Profit-maximizing logistics and pricing strategies in terms of the fares charged and the demand accepted have been developed for the competing transporters.

2. CHARACTERISTIC FEATURES OF CONTAINERIZED TRANSPORT OF GOODS IN INDIA

Containerized Transport

Containerization movement of goods in India has grown significantly in the last few years (see aggregate figures presented in Raghuram [15] and Srivastava [21]). The major modes of land based movement of containers are rail, road and rail-road combinations. A complete analysis of road and rail movements and mode choice issues for containerized movement is beyond the scope of this paper. We mention a few key features of these transport modes that lead to some interesting analytical issues. We restrict ourselves to a few comments on the main factors affecting mode choice and competition between various modes on the inland segment of international containerized traffic.

Trucking Operations

The road sector for freight traffic has been traditionally in the private sector in India, dominated by owners of small sized fleet (often owner-driven trucks). As a result, there is a marketplace at many decentralized locations for transport services to various locations and rates change frequently in response to demands and other operating conditions. For example, queues of vehicles waiting for demand are immediately apparent and pricing decisions can be very easily made as they are formulated and implemented locally.

Rail Operations

In contrast, the rail sector, in this case, including CONCOR (Container Corporation of India) which handles containerized traffic all over the country, has centralized operating policies. The advantage there is a system view of flows is available and cost-effective policies can be designed in the medium run. Here, prices are often negotiated centrally and cannot be changed very quickly.

An Example of Multi-Modal Containerized Transport Operations

A specific example of this scenario is described in Rangaraj and Viswanadham [17] which refers to the flow of export import containers from the aggregation points in North India, (through the Tughlakabad Inland Container Depot in the case of rail) to Jawaharlal Nehru Port Trust (JNPT) in Western India, just south of Mumbai. This example, along with some competitive aspects and a supply chain perspective, is discussed in [17].

In the present paper, an analysis is done on various aspects of such a sector of freight transport. The uniqueness of the analysis is that it seeks to bridge the gap between short term operating policies and long term costs and revenues based on capital utilization on the one hand, and attempts to integrate transport operations strategy of individual operators with economic equilibrium and market share realizations on the other hand.

3. LOGISTICS STRATEGY FOR A TRANSPORTER

We consider a transporter operating a fleet of M vehicles on a network of origins and destinations. The operating strategy of this transporter is described in the scenario of random demand (here assumed to be Poisson for each demand stream from origin i to destination j). The analysis is derived from Nadkarni [13], Sinha [19] and Sohoni [20] which builds on the strategy of a single vehicle and later a fleet of M vehicles of a single operator. The strategy is in terms of empty movements between nodes and refusal of traffic between some location pairs in order to maximize long-run operating revenue.

Notation

The notation used in the model formulation is as follows (for transporter A). In section 4, where a competing transporter B is considered, similar notation is used.

Symbol	Description
n	Number of nodes in the network
i, j, k	Nodes in the network
A	Transporter
λ^0_{ij}	Saturation demand from i to j (tons/hr)
r_{1ij}	Fare at saturation demand from i to j
s_A	A's vehicle capacity (tons/vehicle)
r_{2Aij}	A's fare at cutoff demand from i to j (Rs/ton)
λ_{Aij}	A's demand share from i to j (tons/hr)
r_{Aij}	A's fare from i to j (Rs/ton)
l_{Aij}	A's loaded movements from i to j (veh./hr)
e_{Aij}	A's empty movements from i to j (veh./hr)
q_{Aij}	A's quality of service factor
P_{Aij}	A's share of saturation demand from i to j
t_{Aij}	A's travel time from i to j (hrs)
C_{rA}	A's running cost (Rs/hr)
C_{wA}	A's waiting cost (Rs/hr)
C_A	A's fixed cost (Rs/hr)
w_{Ai}	Number of A's vehicles waiting at i

Demand Acceptance/Refusal

The transporter can decide to accept only a part of the demand and refuse the rest in order to maximize long-run profit. Loaded movements made by a transporter correspond to the demand accepted. The number of loaded movements to be made on each node pair is a decision variable for the transporter. The loaded movements generate revenue for the transporter and also form a part of the cost.

Locational Imbalance in Demand

For an operator with a fleet size M, due to the locational imbalances in the demand, empty movements of vehicles have to be made to preserve flow balance at nodes. The number of empty movements to be made on each node pair is a decision variable for the transporter. These empty movements do not generate revenue. They only add to the cost. For a deterministic demand case, empty movements to balance the fleet size requirements can be modeled as a network flow optimization problem as in Dejax et al [6] and Mishra [11]. A simulation model is described in Ratcliffe [18]. In another context, the cruising behavior of taxis looking for fares on a network is described in Yang and Wong [25].

Temporal Imbalance in Demand

Due to the temporal imbalance in demand, some vehicles might have to wait at nodes. Cost is incurred for waiting. The number of waiting vehicles at each node is found out using Little's law. If each demand stream is a Poisson random variable, the analysis becomes very simple: Each node serves as an M/M/1 queue with memory-less arrivals (of vehicles) with rate θ_i and memory-less service-times (by customers) with rate μ_i. Now, the average length of the queue at node i equals $\dfrac{\theta_i}{\mu_i - \theta_i}$. Assuming that vehicles going empty from any node will not wait i.e. they leave without queuing up, the parameters are

Service rate at node i: $\mu_{Ai} = \sum_j \dfrac{\lambda_{Aij}}{s_A}$

i.e. the total demand (in terms of vehicles/hr) at node i.

Arrival rate at node i :

$$\theta_{Ai} = \sum_j (l_{Aji} + e_{Aji}) - \sum_j e_{Aij}$$

i.e. the difference between the total loaded and empty vehicles coming to node and the total empty vehicles leaving node i.

The number of vehicles waiting at node i is

$$w_{Ai} = \frac{\sum_j (l_{Aji} + e_{Aji}) - \sum_j e_{Aij}}{\sum_j \frac{\lambda_{Aij}}{s_A} - \left(\sum_j (l_{Aji} + e_{Aji}) - \sum_j e_{Aij} \right)}$$

Fleet Size Constraint

This is a resource constraint based on the number of vehicles (fleet size) used by the transporter on the network. The fleet size depends on the investment made by the transporter. The sum of the vehicles running loaded and empty and the vehicles waiting at nodes should equal the fleet size of the transporter.

$$(l_{Aij} + e_{Aij})\, t_{Aij} + w_{Ai} = M_A$$

The planning of fleet size is taken up in Beujon and Turnquist [2].

Factors Affecting the Operator's Profit

Demand Elasticity

The demand on a node pair is a decreasing function of the fare charged. As the fare decreases, the demand reaches a saturation point. The demand cannot increase beyond this saturation point even if the fare decreases further. Demand elasticity has been considered in Sinha [19]. Abdelwahab [1] presents empirical estimates of demand elasticities. The demand is assumed to decrease polynomially with the fare and gradually fall to zero. The demand is assumed to follow a curve as shown.

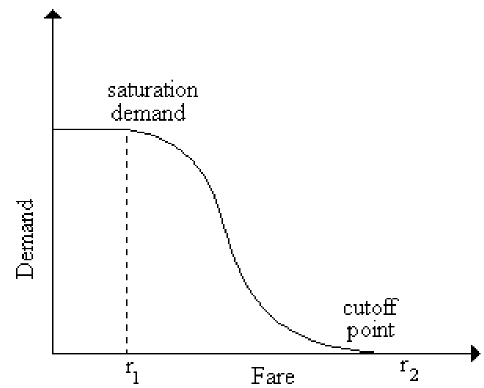

Figure 1
Demand curve

The saturation demand denoted by λ^0_{ij} is the demand at fare = r_{1ij}. Here r_{1ij} denotes the fare, which everybody wishing to transport goods can afford to pay while r_{2ij} denotes the fare beyond which nobody is willing to pay. Therefore r_{1ij} (dependent only on the affordability of the customers) is independent of the transporter's quality of service while r_{2ij} (dependent on the willingness to pay) depends on the quality of service provided by the transporter. Hence r_{2ij} is different for A and B (denoted by r_{2Aij} and r_{2Bij} respectively) while r_{1ij} is the same for both transporters. The model is concerned only with fares between r_{1ij} and r_{2ij} since the transporter will not be able to increase the demand for his/her service by reducing the fare below r_{1ij} and will lose all demand by increasing the fare beyond r_{2ij}.

Operating Costs

The costs incurred by a transporter are the costs of loaded and empty movements, waiting costs, and other fixed costs.

Revenue

Revenue for a transporter is generated by fare charged for the loaded movements. Hence it depends on the demand accepted as well as the fare charged. The empty movements do not generate any revenue.

The results of this analysis (discussed in more detail in Nadkarni [13] and Sohoni [20]) explain the basis for pricing of a single operator in this environment. For example, it explains how the prices charged by an operator on a i-j segment may be different from those charged on the j-i segment, depending on the imbalance of traffic (which necessitates either empty running or waiting, which eventually has to be costed). It includes an assessment of the empty running strategy in the aggregate sense, but also includes the time element of waiting in its analysis. For example, they show that under some assumptions, either *all* demands are accepted or *no* demand is accepted between every pair of locations.

Given the strategies of individual operators, including their pricing decision in the face of demand elasticity, we now model the possible behavior of two competing transport operators for a given stream of traffic, say between nodes i and j. The immediate motivation is to look at the competing modes of traffic mentioned in section 2. The two modes offer different prices and levels of service (in terms of frequency of service - including the inventory consequences of batching of shipments, transit time and convenience factors such as the ease of booking, safety during transit, handling of claims and losses, customs and other commercial formalities). It is seen that there is a resulting market split between modes of transport.

4. COMPETITIVE STRATEGIES

In the presence of competition for transport between different modes of transport, the total demand for transport on the network gets shared depending on the modal characteristics. Competition has been modeled considering fare as the major criterion for mode choice as in and Francois et al [8]. Other factors affecting mode choice have been captured indirectly, through parameters for the quality of service. Network equilibrium has been considered in the models formulated by Yang et al [26], [27] and Zubieta [28].

Factors Affecting Mode Choice

The key factors for mode choice in freight transport in India have been summarized by Cook et al [4] and also discussed in Raghuram [15]. The survey in [4] is based on the Logistics Cost Model of shipper behavior. Both the relative importance of these factors and customer rating of satisfaction have been presented. Mode choice is governed by the commodity characteristics, the customer characteristics and the modal characteristics. It has been found that the most important factors influencing mode choice are reliability, availability, fare, transit time, connectivity, product suitability, loss and damage, negotiability, etc. Factors affecting mode choice switching behavior have been described by Waerden and Timmermans [22]. A joint mode choice – shipment size model formulated by Abdelwahab [1] presents empirical estimates of market elasticities of demand and mode choice probabilities in the freight transport market.

Demand Fulfillment

As in section 3, the demand for transport is been assumed to be Poisson and a demand rate is considered. An operator gets a share of the saturation demand depending on his/her own fare, the fare charged by the competitor and other mode choice characteristics captured by parameters for quality of service. The shared demands are

$$\lambda_{Aij} = \lambda_{ij}^{0}.P_{Aij} \text{ and } \lambda_{Bij} = \lambda_{ij}^{0}.P_{Bij}$$

The total demand $\lambda_{Aij} + \lambda_{Bij}$ may be less than or equal to the saturation demand depending on the fares. Both P_{Aij} and P_{Bij} are functions of the fares r_{Aij} and r_{Bij}. Parameters for the quality of service, q_{Aij} and q_{Bij} denote A's and B's shares of the demand at $r_{Aij} = r_{Bij} = r_{1ij}$. Therefore $q_{Aij} + q_{Bij} = 1$. The higher the value of the parameter, higher is the preference for the transporter's service. In other words, at the fares $r_{Aij} = r_{Bij} = r_{1ij}$, $P_{Aij} + P_{Bij} = 1$. As the fares increase beyond r_{1ij}, the demand falls below the saturation demand and $P_{Aij} + P_{Bij} < 1$. It has been assumed that

$$\frac{q_{Aij}}{q_{Bij}} = \frac{r_{2Aij} - r_{1ij}}{r_{2Bij} - r_{1ij}}$$

since $r_{2Aij} - r_{1ij}$ and $r_{2A\,ij} - r_{1ij}$ denote *A's* and *B's* price ranges and r_{2Aij} and r_{2Bij} depend on the quality of service provided by A and B respectively.

Decision Variables

Each transporter aims to maximize his/her profit and decides the logistics and pricing strategy accordingly. The decision variables are

Pricing decision: Fare between each node pair

Operating decision: The number of loaded movements and empty movements between each node pair

Profit Maximizing Strategies

Both the transporters (A and B) try to maximize their profits without colluding with one another. Williams et al [23] and [24] have discussed the various aspects of competing services, modal choice and the effects of collusion on the optimal strategies. The profit made is a function of the fare charged and the loaded and empty movements made, expressed as the difference between the revenue and the cost.

Costs

The transporters incur running costs (variable costs) denoted by C_{rA} and C_{rB}, waiting costs denoted by C_{wA} and C_{wB} and fixed costs denoted by C_A and C_B respectively. The costs are expressed in Rs./hr. The running costs comprise the fuel and maintenance costs. The waiting costs comprise the cost of waiting at nodes. Various aspects of the costs have been described in Rangaraj and Sohoni [16]. The fixed costs comprise the wages, rents, etc. A detailed analysis of the costs incurred is given in Kumarage [10], Sinha [19] and Sohoni [20]. The total running cost depends on the number of loaded and empty movements and the time of travel t_{ij}. The time of travel is different for different modes of transport as the average running speeds differ. In addition, when two different modes of transport are considered, the distances between a pair of nodes for the different modes too could be slightly different.

The total cost incurred by A is

$$\sum_i \sum_j (l_{Aij} + e_{Aij})\ C_{rA}\ \ t_{Aij} + \sum_i C_{wAi}\ \ w_{Ai} + C_A$$

where w_{Ai} is the number of waiting vehicles at node i (expression derived in section 2).

The total cost incurred by B has a similar expression

Revenue

The revenue [Rs./hr] obtained by a transporter depends on the demand accepted i.e. the loaded movements and the fare charged.

The revenues for A and B respectively are

$$\sum_i \sum_j l_{Aij}\ \ s_A\ \ r_{Aij}$$

where s_A is the vehicle capacity in tons/vehicle.

The total revenue obtained by B has a similar expression

Profit Earned

The profit earned is the difference between the revenue generated and the costs incurred.

Constraints

Flow balance constraints: These constraints have to be satisfied at each node in the network, since the number of incoming and outgoing vehicles has to be equal at each node.

$$\sum_j (l_{Aji} + e_{Aji}) - \sum_j (l_{Aij} + e_{Aij}) = 0 \quad \forall\ i$$

The flow balance constraint for transporter B has a similar expression.

Fleet size constraints:

Let N_A denote the fleet size of transporter A. Then the fleet size constraint is given by

$$(l_{Aij} + e_{Aij})t_{Aij} + w_{Ai} = N_A$$

The fleet size constraint for transporter B has a similar expression.

Capacity constraints

These number of loaded movements should be less than or equal to the demand share.

$$l_{Aij} \le \frac{\lambda_{Aij}}{s_A}$$

The capacity constraint for transporter B has a similar expression.

Non-negativity constraints

All the decision variables should be non-negative.

$$r_{Aij},\ r_{Aij},\ l_{Aij},\ l_{Bij},\ e_{Aij},\ e_{Aij} \ge 0$$

Optimal Strategies

Transporter A aims to maximize the profit G_A subject to constraints at the existing strategy of B. Here r_{Aij}, l_{Aij} and e_{Aij} are the decision variables and r_{Bij}, l_{Bij} and e_{Bij} are constants.

Thus A's strategy is as follows

$$\max \ \sum_i \sum_j l_{Aij}\ \ s_A\ r_{Aij} - \sum_i \sum_j (l_{Aij} + e_{Aij})\ C_{rA}\ \ t_{Aij}$$
$$-\sum_i C_{wAi}\ \ w_{Ai} - C_A$$

s.t.

$$\sum_j (l_{Aji} + e_{Aji}) - \sum_j (l_{Aij} + e_{Aij}) = 0$$
$$(l_{Aij} + e_{Aij})t_{Aij} + w_{Ai} = M_A$$
$$l_{Aij} \le \frac{\lambda_{Aij}}{s_A}$$
$$r_{Aij},\ l_{Aij},\ e_{Aij} \ge 0.$$

For every value of r_{Bij}, l_{Bij} and e_{Bij}, for all i and j, a set of values r_{Aij}, l_{Aij} and e_{Aij} is obtained.

This represents a curve corresponding to A's strategy in the space described by r_{Aij}, l_{Aij}, e_{Aij} r_{Bij}, l_{Bij} and e_{Bij}.

Similarly transporter B aims to maximize the profit G_B subject to constraints at the existing strategy of A. Here r_{Bij}, l_{Bij} and e_{Bij} are the decision variables and r_{Aij}, l_{Aij} and e_{Aij} are constants.

Transporter B's strategy is similar to transporter A's strategy.

For every value of r_{Aij}, l_{Aij} and e_{Aij}, for all i and j, a set of values r_{Bij}, l_{Bij} and e_{Bij} is obtained.

It represents a curve corresponding to B's strategy in the space described by r_{Aij}, l_{Aij}, e_{Aij} r_{Bij}, l_{Bij} and e_{Bij}.

The intersection of the curves corresponding to A's and B's strategies gives the equilibrium point. The solution methodology and a numerical example are available in Moharir [12].

Game Theoretic Equilibrium

In game theoretic parlance, the competition between two different modes of transport on a transportation network is an instance of a two-person, non-cooperative, non-zero sum game. The strategy for a transporter is in terms of the fare charged and the loaded and empty movements made provided the competitor's strategy is known. Equilibrium is attained when neither of the two transporters can gain by deviating from his/her strategy as long as the opponent's strategy remains fixed. The equilibrium point gives the optimal strategies of the two transporters.

The service level as a result of operating policies would be different in different scenarios. For example, in one scenario, timed movements of vehicles would result in a frequency of service. In another scenario, a given fleet size and an operating policy would result in a certain availability of vehicles at nodes for different streams of traffic. This results in waiting times, derived from queuing theory, which could be translated to service levels. These need to be elaborated in different cases.

5. CONCLUSIONS

Network operating policies for transport have earlier concentrated on specific aspects such as movement of empty vehicles (in freight train operations or taxi operations), deciding on frequency of services (for example in passenger bus operations) and fleet size planning. Separately, the impact of pricing on demand has been considered, mostly in the transport economics literature. A further strand in the literature is about the factors affecting mode choice of shippers where alternate transport options are available. We attempt the task of providing an integrated view of these aspects. With given demands on a network, we first provide an operating policy, which includes waiting costs and empty running costs (by allowing for the strategy of refusal and empty running). We then include demand elasticity considerations of pricing in an overall revenue maximization for a single operator with a fleet of vehicles. Finally, we model competition between two operators for a market of some size, and establish an equilibrium set of prices and operating policies.

The discussion in the paper also attempts to provide a basis for explanation of phenomena such as asymmetric fares or prices between pairs of locations, behavioral aspects of transport operators such as taxis and some pricing policies of larger transporters.

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