Matching and allocation problems as part of larger decisions

Narayan Rangaraj

Industrial Engineering and Operations Research

Madhu N. Belur

Department of Electrical Engineering

IIT Bombay

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Outline

- Part 1 - Stand-alone problems
  - Examples
  - Objectives
  - Constraints
  - Methods

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  - Methods
- Part 2 : Matching and allocation as part of larger problems
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Part 1: Stand alone problems

Various matching requirements

- Allocation: preferences (preliminary)
- Assignment: final: best or meeting various requirements
- Sometimes: needs to rule-out complaints (‘stable matching’)
- Sometimes hard constraints: class-room capacity constraints
- Often soft constraints: incorporate through a term in the objective
- Different objectives: ‘total satisfaction index’

Some examples

- Students with a rank $\leftrightarrow$ Institute-programmes
- In a department: instructors $\leftrightarrow$ courses
- Students $\leftrightarrow$ thesis-advisors
- Timetabling:
  Instructor-course $\leftrightarrow$ time-slots $\leftrightarrow$ classrooms

Large national-level exams

GATE or JEE: Indian National level exams

1. Candidates $\leftrightarrow$ exam-centers
2. Invigilators $\leftrightarrow$ exam-centers
3. Post-exam: Admissions: candidates $\leftrightarrow$ institute-programmes
4. The first two above: not critical: just total satisfaction to be maximized
5. Admissions: critical: 'fairness' in allotment

Fairness in admission/stable marriage

1. A match between Men M and Women W is called stable if:
   for **no** two matched pairs (m_i, w_i) and (m_j, w_j), it happens that:
   m_i prefers w_j more than w_i, and also w_j prefers m_i more than m_j
2. Admissions: we call an allotment of candidates C to programmes P as 'fair' (or complaint-free) if:
   for **no** two allotted pairs (c_i, p_i) and (c_j, p_j) it happens that
   c_i prefers p_j more than p_i, but c_j has a lower rank than c_i

When candidates can have multiple ranks (in say multiple papers), then complaint-free (or fair) is same as stable-marriage

JAM seat allotment

Joint Admission to MSc (JAM) (across 15 IITs: 70 course-programmes)

- Each candidate can write one or two out of 7 papers
- Many candidates have multiple ranks
- Course-programme preferences is single ordered list per candidate
- Due to volume of candidates, there are ties in marks, and hence multiple candidates per rank: about 2 to 3 candidates per rank (at most)
- Capacities provided for each course-programme: no tie-breaking rule. Extra seat if tie at the last rank
- Assignment: stringent requirements: no manual over-ride: high answerability
- About 1000 seats and 5000 ranked candidates
- 'Bucket-filling' not possible - candidates have ranks in multiple papers

We implemented a stable-marriage with 'seat-increment-at-rank-tie-at-last-seat' feature

Course Teaching Assistant ↔ Courses

Typically: 90 courses+labs and 500 Course TAs
Manual over-ride/change allowed/possible
Complex requirements, but not ‘critical’ assignment: just one semester
Trade-off between TA satisfaction and course/instructor satisfaction

- Course: every course (instructor) prefers high-GPA TA
- Students/TAs: want higher choice course
- Administrator (like Dept course coordinator):
  - Large courses have multiple TAs: a ‘mix’ of high/low GPA TAs
  - Variance of the GPA across courses needs to be small: to equalize TA-skills across courses

Non-convex performance objectives on a matching
Set of matchings very large: $n!$
Markov-Chain Monte-Carlo: hop from matching (state) to ‘neighbour’ state
Need to ensure irreducibility and rapid-mixing

Candidates $\leftrightarrow$ Interview Committees

Many PhD candidates visit IIT Bombay for admission to PhD

After written test (for shortlisting) on one day, interviews are to be held the next day

Allotment to interview-committees: allot 140 candidates to 15 committees

- Allotment not critical: manual over-ride/change possible
- If 90% allotment is done quickly, interviews can begin, and rest can be manually allotted
- Candidates give preferences of areas-of-research
- Committees are composed of members having areas-of-research
- Need to allot candidate to committee with most overlap of areas of research (and preferably better choice of candidates' areas)
- Need to 'normalize' interviewing load across committees

After suitable reward matrix (using committee-areas' overlap and student preferences), we used a standard assignment algorithm

Summary of Part 1

- Many different decision settings
- Criticality of decision and manual over-rides
- Methods are varied
- Local constructive procedures - very useful when local feasibility conditions are to be verified/satisfied
- Linear programming based formulations - useful when global information is available in convenient form
- Randomized search for more complicated objectives - quite robust and give good solutions

Part 2 : Allocation as part of larger problems

Three examples

- University timetabling - bottleneck allocation resources planned separately and they are taken as inputs in a bigger schedule
- Railway platform allocation - global aggregate constraints often result in feasible resource allocation
- Rolling stock assignment - short time frame assignments as part of long time horizon optimisation

University timetabling

- Students have certain courses to credit
- Courses need to be assigned time-slots
  - Instructors known - instructor constraints to be respected
  - Student clashes to be minimized
- Course slot combinations to be assigned to rooms
  - Room capacity constraints
- Other considerations such as minimizing student movement
- Too large a problem to be solved in one go
- Only a few slots need central room allocation
- Iterative procedure with room allocation done at a different level in the procedure
- Allows for localized decision making also

Railway platform allocation - as part of timetable construction

- Overall problem - fix timings of several services to meet various requirements
- Stringent occupancy and vacation constraints for platforms
- Three linked allocation problems
  - Arrival events of services to be allotted platforms
  - Departure events of services to be allotted platforms
  - Arriving events to be linked with departure events
- Quite difficult to solve together with timing variables (many integer variables, and large numbers needed to link continuous variables with integer variables), BUT
- Each of these easy to solve if the sequence of events is known
- Slight changing of arrival/departure timings seem feasible at first sight
- Eventually solved in sequential iterative procedure

Summary of rail example

- Focus on planning resources/activities in the peak period - can easily be extended to off peak periods
- About 50 train units (12 car rakes each carrying almost 4000 people)
- About 300 services in the peak period on a subnetwork - Mumbai harbour line network
- 1500 events that have to be modelled (timings to be decided)
- Platform allocation in the peak period - 45 arrivals and departures - rake linkages, 45 arrival events, 45 departure events
- These have to be done in all terminals, but need to be done in detail only at a few terminals (one terminal?)
- We have tried both periodic and aperiodic versions of the timetabling problem involving traversal, halts, service frequencies, rolling stock planning, platform allocation and terminal considerations
- Large problem involving several hundred integer variables and a few hundred timing variables, but if solved sequentially and iteratively, turns out to be manageable

Local allocation decisions

- Linkages - easy rules, first in first out, subject to (for example)
  - 3 minute minimum turnaround
  - 5 minute maximum turnaround, but that is automatically achieved
- Platform allocation for arrivals
  - For two platforms - platform 1 if available, else platform 2.
  - Similar rules for multiple platforms
- Easy to implement in local, constructive procedures, not so easy to include with integer constraints, especially if part of larger problems

Rolling stock assignment

Rolling stock assignment - short time frame assignments as part of long time horizon optimisation

- Basic requirement: allot vehicles (railway rakes) to demands on a daily basis - on any given day, vehicles available at various points and demands at various points - an allocation decision has to be made
- Long term requirement: each rolling stock item needs to visit a maintenance facility 'regularly'
- Need to have a daily schedule that also respects this longer term (typically a month) requirement
- With (largely) predictable demand, global calculation tells us whether local allocations are likely to be optimal

Allocation as part of larger problems

- If allocation to be done as part of a larger decision, there are options as to how to handle this
- Integrated approach is to include all variables in the master problem
  - Allocation variables appear in only a subset of constraints and with very few linkages to other variables
- Makes search cumbersome, even though modern solvers could discover some of these structures
- Pure sequential solution, where allocation decisions are taken later, after fixing some other variables
  - Often suboptimal, sometimes even infeasible
- Iterative sequential solution
  - Fix outer variables and solve allocation problems in an inner loop and iterate if needed - this is viable and close to optimality

Summary of IIT Bombay and IEOR@IIT Bombay

- IIT B established in 1958
- Has all major engineering and science departments, management, design, humanities and several interdisciplinary groups and programmes
- About 10000 students, historically strong undergraduate students, but now more graduate students than undergraduate students
- More than 3000 Ph.D. students
- Close to 700 faculty members
- Visitors, including researchers, short and medium term exchange students are welcome

- Industrial Engineering and Operations Research (IEOR) at IIT Bombay
- Established in 1978 - now 10 faculty members, 100 students, all postgraduate
- Work in the areas of Optimisation, Game Theory, Stochastic Processes and Applied Probability, Simulation, Machine Learning and several application areas, including transportation, logistics, health care, manufacturing and finance