IEOR, IIT BOMBAY: M.Tech Admissions 2025 Test for RA/RAP Candidates Read instructions below carefully

- You may refer to any material/book/notes for the exam. Kindly do NOT consult or collaborate with any person for this test.
- Answers to the questions of this test are to be submitted using the Google Form sent to your email. Your submission will be considered during the Interviews. You may be asked to show your workings during the interview.
- You can write your answers on plain sheets of paper. Each page should have your name AND application ID written on the top of the page. Question numbers should be clearly written.
- To submit your answers, scan/take photo of your answer sheets and upload a single pdf file via the online Registration Form.
- In case you write the answers using Word/Latex/etc, then you can export into pdf file and upload the pdf via the Registration form.
- There are **7** questions in this test.
- Please note that a range of questions has been asked, considering the range of applicants and projects available. Do attempt as many questions as you can.
- All questions are self explanatory. If you have doubts, please make suitable assumptions and include those assumptions in your answers. Please do not contact the Admissions Committee for clarification on the questions.
- Deadline for submission is 23:59 hours, June 04, 2025.

Questions

- 1. Draw the feasible region of the two-dimensional set $S = \{x \in \mathbb{R}^2 | x_1 x_2 \ge 1, 25x_1^2 + 100x_2^2 \le 400\}$. What is the minimum value of $x_1 + x_2$ in this set? Explain your approach.
- 2. (SJ) A robot operates in a warehouse represented by a 6×6 grid. Each cell is one of the following:
 - 0 Free space (movement cost = 1)
 - 1 Obstacle (impassable)
 - C Congested zone (movement cost = 3)
 - $\bullet~S-Start$ position
 - $\bullet~G-Goal$ position

The robot starts at S and must reach G, moving only up, down, left, or right (no diagonal moves allowed). The grid is given as:

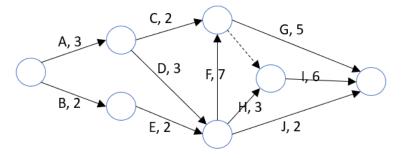
S	0	1	0	С	G
1	С	1	0	1	1
0	0	0	0	С	0
1	1	С	1	0	0
0	0	0	0	С	1
1	1	1	0	0	0

Answer the following:

- (a) Find the minimum-cost path from S to G.
- (b) Show the path step-by-step and the cost incurred at each step.
- (c) If the movement cost through a congested zone (C) increases from 3 to 6, does the optimal path change? Justify briefly.
- 3. A fulfillment center processes online orders that require workers to pick items from shelves and move them to the packing zone. On average, it takes a worker 45 minutes to complete a full pick-pack cycle, including walking, handling, and occasional scanning delays. Additionally, workers take a 15-minute break after every 3 cycles.
 - (a) The manager wants to dispatch one worker into the picking area every 5 minutes during a 16-hour operational window to maintain continuous throughput. Estimate the minimum number of workers needed to support this dispatch schedule.
 - (b) A supervisor proposes halting order picking between 2 a.m. and 6 a.m. for restocking and maintenance. Will this off-hour break lead to a reduction in the number of workers needed for maintaining the same dispatch frequency during active hours? Explain your reasoning.
- 4. Consider a real-valued function, f(x̄) ∈ [0, 1], defined on the bit-strings, x̄, of length N = 64.
 (a) Draw a schematic plot showing, f(x̄) vs. x̄, assuming f(x̄) values are uniformly randomly distributed between .1 and .9.

(b) From the plot in (a) identify at most 3 local minima and at least one global minimum. (c) Argue whether or not stochastic optimisation algorithms based on gradient descent will work for finding the global minimum of the function $f(\bar{x})$ in your particular case.

- 5. A manufacturing process is currently following a screening procedure described below. It is known that 10% of the units are defective. The screening procedure used will pass a non-defective unit with probability 1, while a defective unit has a probability of 0.1 of being passed. A unit, selected at random, was subjected to the screening procedure, and it passed. What is the probability that it is defective?
- 6. Consider the project network given below with project-activities denoted on the arcs along with their durations. Identify ALL the critical activities of this project.



- 7. Patients are given appointments for meeting a doctor at a certain speciality clinic. In one of the suggested schemes for making appointments, 6 patients are called at 8:30AM. This scheme helps in reducing the idle time of the doctor (i.e., after meeting a patient, the doctor does not have to wait for the next patient to arrive), but all patients except the first one have to wait for their turn. Suppose the meeting time of each patient with the doctor is a uniform random variable ranging from 3 to 15 minutes, and assuming that meeting times are independent of each other, answer the following two questions
 - (a) What is the waiting time distribution of the second patient that the doctor sees? Explain.
 - (b) What is approximately the waiting time distribution of the 6th patient? Explain.

You may assume that all patients arrive simultaneously at the scheduled time of 8:30AM.