Evolutionary Vaccination Games with premature vaccines to combat ongoing deadly pandemic

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Joint work with

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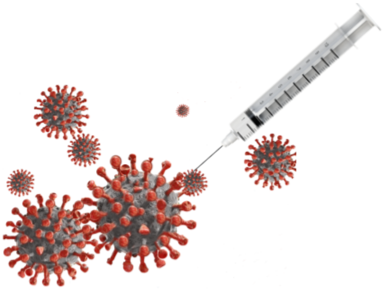
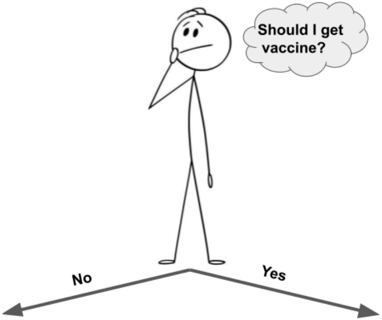
Outline

Motivation

- today, living through a pandemic, COVID-19
- disease characteristics are unknown
- pre-mature vaccines are being introduced
- lack of information, e.g., possible end of disease
- vaccination responses of others matter
- decision about vaccination needs to be made





Vaccination hesitancy

- lack of faith in the efficacy
- emerging fears due to the reported side effects

Vaccination urgency

- reported information about the sufferings and deaths due to disease
- lack of hospital services

Vaccination game

- dynamic behavioural vaccination responses of population
- choice between two contrasting fears - fear of vaccine, fear of infection
- voluntary vaccination leads to evolutionary game theoretic framework

Objectives

- to study the dynamics and understand the equilibrium states
  - based on disease parameters
  - based on population vaccination responses
- to identify the vaccination responses that are stable against mutations
  - evolutionary stable (ES) equilibrium states

Literature

In [1], [2] seasonal variations are considered

- vaccination season: vaccinate and pay a fixed cost, or try free riding
- disease season: infected pay infection cost
- replicator dynamics based models, learn from previous seasons

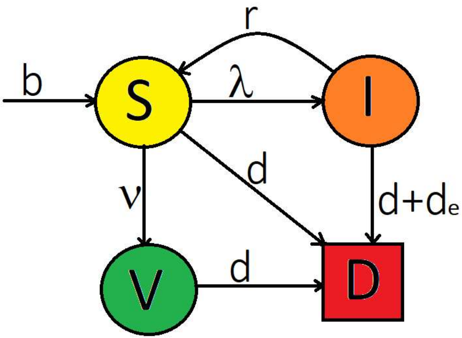
In [3], vaccinate before the outbreak or on any week of the pandemic or never

- one among the 53 weeks is chosen, 53 being never vaccinate
- early vaccination or free-rider, ‘wait and see’ equilibrium

- in majority of literature, some information about disease is assumed
- e.g., time duration and time of occurrence of disease is known
- we do not assume any such information

SIS model with vaccination

- $N(t) = S(t) + V(t) + I(t)$
- $\theta(t) := \frac{I(t)}{N(t)}$, $\psi(t) := \frac{V(t)}{N(t)}$, and $\phi(t) := \frac{S(t)}{N(t)}$
- $b > d + d_e$



Vaccination responses

- vaccination response depends on $(\theta(t), \psi(t))$
- probability of vaccination is $q(\theta, \psi) = \min\{1, \tilde{q}(\theta, \psi)\}$
- **vaccination policy:** $\pi(\beta)$

Follow-the-crowd (FC)

$$\tilde{q} := \beta\psi(t)$$

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Free riders (FR)

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Vigilant follow-the-crowd (VFC1)

$$\tilde{q} := \beta\theta(t)\psi(t)$$

Dynamics

- at $(k+1)^{th}$ transition epoch

$$I_{k+1} = I_k + \underbrace{\mathbb{I}_{k+1}}_{\text{new infection}} - \underbrace{\mathbb{R}_{k+1}}_{\text{new recovery}} - \underbrace{\mathbb{D}_{I,k+1}}_{\text{new death}},$$

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- stochastic approximation iterates for $\theta_k = I_k / N_k$

$$\theta_{k+1} = \theta_k + \epsilon_k \frac{1}{\eta_{k+1}} \left[\mathbb{I}_{k+1} - \mathbb{R}_{k+1} - \mathbb{D}_{I,k+1} - \Delta N_k \theta_k \right], \quad \epsilon_k := \frac{1}{k+1}$$

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$$V_{k+1} = V_k + \underbrace{\mathbb{V}_{k+1}}_{\text{new vaccination}} - \underbrace{\mathbb{D}_{V,k+1}}_{\text{vaccinated dies}}$$

- stochastic approximation iterates for $\theta_k = I_k/N_k$

$$\theta_{k+1} = \theta_k + \epsilon_k \frac{1}{\eta_{k+1}} \left[\mathbb{I}_{k+1} - \mathbb{R}_{k+1} - \mathbb{D}_{I,k+1} - \Delta N_k \theta_k\right], \quad \epsilon_k := \frac{1}{k+1}$$

- similarly, one can write stochastic approximation iterates for ψ_k, ϕ_k and $\eta_k = N_k/k$

Stochastic approximation induced ODE[5]

$$\dot{\theta} = \frac{1}{\eta\varrho}\left[\theta\phi\lambda - r\theta - b\theta - (d_e - d_e\theta)\theta\right], \;\; \phi = 1 - \theta - \psi$$
$$\dot{\psi} = \frac{1}{\eta\varrho}\left[q(\theta,\psi)\phi\nu - (b - d_e\theta)\psi\right], \;\; \text{and,} \tag{1}$$
$$\dot{\eta} = \frac{b - d - d_e\theta}{\varrho} - \eta, \quad \varrho = (b + d + d_e\theta + \lambda\theta\phi + \nu\phi + r\theta).$$

Theorem

Under some assumptions:

1. *the sequence* $(\theta_k, \psi_k, \eta_k)$ *converges to attractors of the ODE*
2. *dynamics of the process can be approximated using the ODE solutions*

Main Observations about equilibrium state

With no excess deaths $d_e = 0$: $(\rho := \lambda/(r+b+d_e),\ \mu := b/\nu)$

- self-eradicating disease $(\rho < 1)$[1] gets eradicated without vaccination, $(\theta, \psi) = (0, 0)$
- endemic disease $(\rho > 1)$:
  - vaccination hesitancy $\rightarrow$ non-vaccinated disease fraction, NVDF $= \left(1 - \frac{1}{\rho}, 0\right)$
  - vaccination urgency at equilibrium $\rightarrow$ coexisting equilibrium

$$(\theta_E, \psi_E) := \left(1 - \frac{1}{\rho} - \frac{1}{\mu\rho},\ \frac{1}{\mu\rho}\right)$$

  - vaccination urgency from beginning $\rightarrow (0, \psi_f)$ disease is eradicated!
  - not possible to eradicate the disease with vigilant agents

[1] $\lambda - r - d_e < b$

Numerical simulations: ODE attractors versus actual system

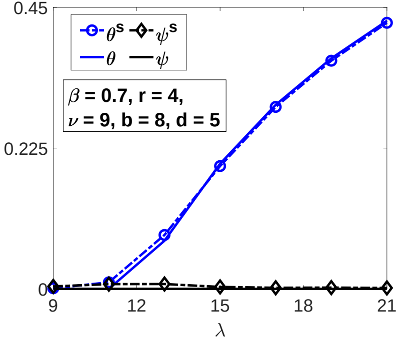

Figure: FC agents vs λ

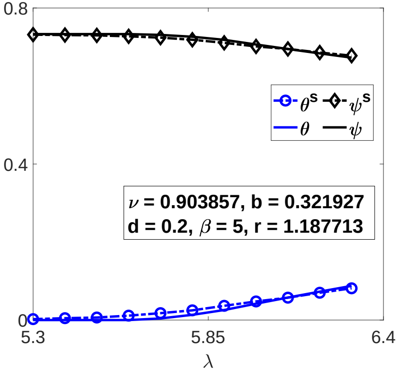

Figure: FR agents versus λ

- $d_e = 0$

Equilibrium for deadly disease ($d_e > 0$)

- conjectured attractors for follow the crowd and free-riders
- self eradicating disease leads to $(0, 0)$
- for endemic disease, let $\rho_e := {}^{\lambda - d_e}/_{r+b}$, $\mu_e := {}^{b-d_e}/_{\hat{\beta}\nu - d_e}$
  - vaccination hesitancy $\rightarrow$ NVDF $\left(1 - \frac{1}{\rho_e}, 0\right)$
  - vaccination urgency at equilibrium $\rightarrow$ coexisting equilibrium $(\theta_E^{d_e}, \psi_E^{d_e})$

$$(\theta_E^{d_e}, \psi_E^{d_e}) \approx \left(1 - \frac{1}{\rho_e} - \frac{o^{d_e}}{\mu\rho_e}, \frac{o^{d_e}}{\mu\rho_e}\frac{\lambda - d_e}{\lambda}\right) \text{ with } o^{d_e} := \frac{1}{1 + \frac{d_e(r+d_e-\lambda-\nu)}{\mu\lambda\nu}}$$

Evolutionary behaviour

- for any $\pi(\hat{\beta})$, system reaches an equilibrium
- what if mutants invade a system in equilibrium?
  - is original vaccination response still better?
  - does the equilibrium drift away?
  - which equilibrium states are evolutionary stable?
  - which policies lead to evolutionary stable (ES) states?

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ESS-AS : Evolutionary Stable Strategies Against Static mutations

A policy $\pi(\beta)$ is said to be ESS-AS if

1. if $\{q_{\pi(\beta)}\} = \arg\min_{q \in [0,1]} u(q, \pi(\beta))$,
2. there exists an $\bar{\epsilon}$ such that $\{q_{\pi(\beta)}\} = \arg\min_{q \in [0,1]} u(q, \pi_\epsilon(\beta, q'))$, for any $\epsilon \leq \bar{\epsilon}$

Anticipated user utility at equilibrium

cost of vaccination

$$\underbrace{c_{v1}}_{\text{fixed cost}} + \underbrace{\min\left\{\bar{c}_{v_2}, c_{v_2}/\hat{\psi}\right\}}_{\text{perceived cost due to side effects}}$$

cost of infection

$$\underbrace{p_I(\hat{\theta})}_{\text{P(infection)}} \left(\underbrace{c_{I_1}}_{\text{infection cost}} + \underbrace{c_{I_2} d_e \hat{\theta}}_{\text{cost of death}} \right)$$

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utility of policy q against population profile $\pi(\hat{\beta})$:

$$\begin{aligned} u(q;\pi(\hat{\beta})) &:= q \times \text{cost of vaccination} + (1-q) \times \text{cost of infection} \\ &= q \times \Delta + \text{cost of infection, where} \\ \Delta &= \text{cost of vaccination} - \text{cost of infection} \end{aligned}$$

ESS-AS

Non-vaccinating ESS

- if the disease is self eradicating $\rightarrow$ (0,0), or
- if anticipated cost of vaccination ($\Delta > 0$) is more at NVDF $\rightarrow (1 - {}^1\!/\!{\rho}, 0)/(1 - {}^1\!/\!{\rho_e}, 0)$

- in some cases, there is no ESS at all
- ESS is either possible in all vaccination responses, or in none!!!

ESS-AS

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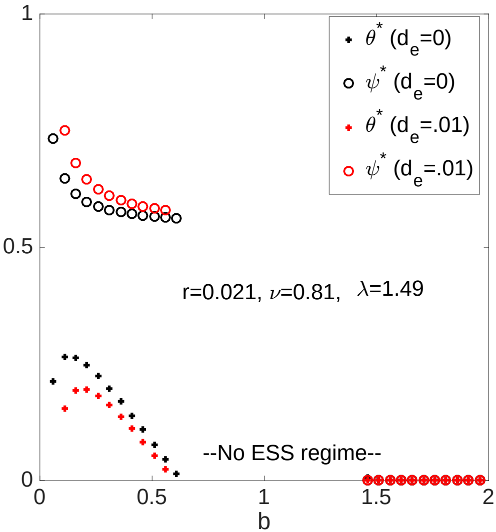
Vaccinating ESS

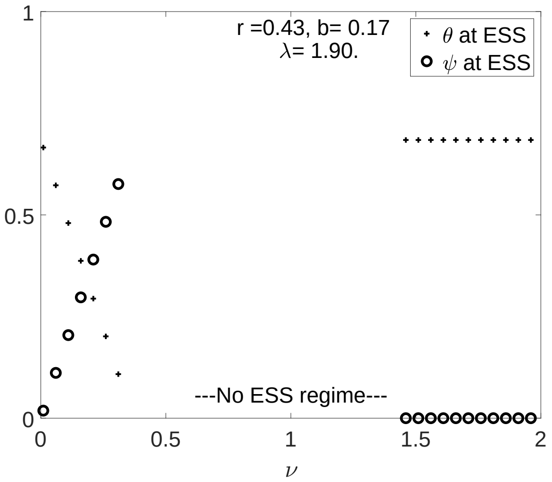
- probability of vaccination is 1 at equilibrium
- vaccination response leads to co-existing equilibrium (θ_E, ψ_E) / $(\theta_E^{de}, \psi_E^{de})$
- anticipated cost of infection at corresponding equlibrium is high ($\Delta < 0$)

ESS versus birth-rate

- with excess deaths,
  - higher vaccinated fraction
  - lower infected fraction
  - pattern is similar
- for small birth-rate
  - higher infection rate per birth
  - higher vaccinated fraction
  - lower infected fraction as ESS
- for large birth-rate, self eradicating disease



ESS versus vaccine availability



- infected ↓ and vaccinated ↑ initially
- might expect high vaccinated with more vaccine availability
- the converse is true !!
- perception towards infection cost changes with abundance of vaccine
- non-vaccinating ESS leads to NVDF

Conclusions

- motivated by COVID-19 pandemic, prematurely introduced vaccines
- derived infected/vaccinated proportions for various behavioural patterns
- many vaccination responses eradicate the disease
- disease is never eradicated under ES policies, unless self eradicating
- different dynamics for different response, but ES state is same
- ESS either exists in all behaviours or in none
- abundance of vaccine leads to lower vaccinated proportion
- ironically, disease can be better curbed with excess deaths

References

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Thank you